Kinks and Gains from Credit Cycles*

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Abstract

Credit-market imperfections are at the centre stage of several theories of business fluctuations. We revisit the fundamental task of quantifying the cost of business cycles in a model where household borrowing is subject to a collateral constraint. Business cycles affect the degree of credit tightness, thus making households financially unconstrained, from time to time. We show that this effect overcomes the conventional losses from uncertainty, thus making fluctuations welfare-dominate certainty. Our findings imply that evaluating the desirability of macroprudential policies solely based on their ability to reduce macroeconomic volatility may be misguiding.

Keywords: Cost of business cycles, collateral constraints, precautionary saving.

JEL codes: E20, E32, E66.

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1 Introduction

Much macroeconomic research is devoted to examining business cycles and designing stabilization policies that could possibly attenuate these. To advocate a role for such policies, it does not seem unreasonable to identify relevant welfare costs entailed by the business fluctuations one suggests to dampen (Lucas, 1987). In this paper, we seek to quantify the cost of business cycles through the lens of a model featuring collateralized borrowing. This is a relevant task, for two main reasons. From a practical viewpoint, the last decade has witnessed a formidable resurgence of interest in the role of credit-market frictions for business fluctuations. Likewise, the demand for macroprudential policies capable of dampening fluctuations arising from, or magnified by, credit markets is high on the policy agenda. From a modeling viewpoint, collateral constraints are key to generating sizeable asymmetries in economic activity, exacerbating cyclical downturns and smoothing upturns by tightening or relaxing credit conditions (see, e.g., Guerrieri and Iacoviello, 2017 and Jensen *et al.*, 2020). Our key contribution is to show that the asymmetric business fluctuations arising in such a context may be welfare-improving.

The usual qualitative argument for the emergence of business-cycle costs takes as a starting point a concave and continuous welfare function of some state of the economy. Thus, one compares the outcome from the deterministic case with its counterpart in the stochastic case, where the state fluctuates around the same deterministic value. By Jensen's inequality, the former outcome is preferred. While Lucas's seminal contribution did not criticize this qualitative argument, it strongly questioned its quantitative relevance. Using a conventional CRRA utility function and a statistical model of consumption, he found losses as low as 0.008% of consumption.[1]

Various contributions have considered collateral contracts à la Kiyotaki and Moore (1997) as a way of enforcing debt repayments from borrowers to lenders. Accordingly, it is assumed that individuals and firms cannot borrow more than a fraction of the value of a certain collateral asset. This feature may be quantitatively important in that it generates a financial accelerator effect, according to which shocks to the economy are propagated through movements in the collateral value. In light of this, one might expect the salience of collateral

[1]This negligible size has since been contested by a vast literature identifying higher welfare costs of fluctuations in extensions of Lucas's simple framework. These include, but are not limited to, models with incomplete financial markets (İmrohoroğlu, 1989; Krusell and Smith, 1999; Krusell *et al.*, 2009; Storesletten *et al.*, 2001), imperfect competition (Galí *et al.*, 2007), detailed time-series modelling of consumption (Reis, 2007; De Santis, 2007), non-expected utility functions (Obstfeld, 1994), asset prices (Alvarez and Jermann, 2004), endogenous growth (Barlevy, 2004), and disaster risk (Barro, 2006). While some significant increases have been recovered, consensus still seems to be that the cost is small.

constraints to lie in their amplification of business cycles and, therefore, the associated welfare cost. We show that, despite their fundamental role as propagators of business fluctuations, collateral constraints induce non-linearities in borrowers' economic decision rules that have the potential to make uncertainty welfare enhancing, all else equal, relative to a state where collateral constraints bind deterministically.

Collateral constraints introduce a discontinuity that plays a central role: They are either binding—the so-called 'constrained regime'—or not—the 'unconstrained regime'. Policy functions will therefore feature a *kink* at the point where the model switches from one regime to the other. Starting from this simple consideration, we calibrate a small-open economy where borrowing is subject to a collateral constraint, with the underlying collateral asset being represented by the available stock of housing. In this setting, we find that business cycles can be *beneficial* for welfare, with the benefit being one order of magnitude larger than Lucas's number, in absolute value: Fluctuations in credit conditions and income *raise* unconditional welfare by around 0.21% of quarterly consumption in perpetuity. As agents in the domestic economy are more impatient than international lenders—and therefore prone to borrowing—the steady state of the model is characterized by a binding credit constraint.[2] This source of inefficiency restricts consumption below the level attainable if agents were able to act as standard consumption smoothers—thus reducing their lifetime utility. Gains emerge as, being subject to fluctuations, the economy displays episodes of non-binding credit constraints that temporarily alleviate the key source of inefficiency in the economy.

To understand the driving factor behind our result, it is convenient to first reconsider the key insight of Lucas (1987). Here, the emergence of a cost of business fluctuations depends on the interplay between uncertainty and consumers' prudence, as embodied by the convexity of their marginal utility of consumption. In the presence of risk aversion, households prefer a stable consumption path, as compared with one that fluctuates around the same mean. As a result, some compensation would be necessary to make consumers indifferent between the two consumption paths. This well-known mechanism—which will be referred to as the *fluctuations effect*—is at play in our model of dynamic consumption-saving decisions, as we employ a standard utility function exhibiting prudence (Kimball, 1990), where uncertainty about income and financial conditions leads to precautionary saving.

The introduction of a financial constraint induces an additional precautionary motive, as

[2] This is customary in the long-standing tradition of dynamic stochastic general equilibrium models with credit constraints, ever since Kiyotaki and Moore (1997).

consumers try to reduce the risk of finding themselves financially constrained. The arrival of shocks, combined with households' decisions, endogenously determine whether the credit constraint binds or not. The resulting kink in debt determination is crucial, as it facilitates temporary switches to a regime in which the constraint does not bind, allowing households to smooth consumption, from time to time. In this respect, fluctuations have an advantageous effect on welfare, as they induce occasional switches to an 'efficient' regime. In the remainder, we refer to this as the *endogenous switching effect*. In our calibrated model, we find this effect to be stronger than the fluctuations effect, thus paving the way for business cycles to entail a gain.

In the existing literature, the hypothesis that business fluctuations could be welfare enhancing has rarely been put forward. In this respect, one exception is Cho *et al.* (2015), who find that gains from business cycles may arise in a conventional real business cycle model where labor demand is a convex function of productivity shocks. The presence of multiplicative shocks is crucial for this result: Such shocks have the potential to raise the mean level of output and/or consumption, allowing agents to take advantage of uncertainty by working harder and investing more during expansionary periods.[3] By contrast, when uncertainty enters the economy additively, it has no beneficial effect on the choices that can be adjusted to it. While a *mean effect* of uncertainty is at play also in our setup, the endogenous switching effect operates independently and produces gains from business cycles, even when only additive shocks are at play.

Our paper is also related to recent work by Jordà et al. (2020), who show that once the negative skewness of consumption growth in the data is accounted for, the welfare cost of fluctuations increases substantially. They reach this result using a statistical model of consumption where the certainty benchmark features no inefficiencies. By contrast, while our model also matches the skewness in consumption growth observed in the data, our certainty benchmark is a distorted steady state in which the presence of a binding borrowing constraint leads to an inefficiently low level of consumption.

Our findings have important implications for the assessment and the conduct of economic stabilization policies. In contexts where financial constraints have the potential of becoming slack, from time to time, evaluating the desirability of such policies solely based on their

[3] As discussed by Fernández-Villaverde and Guerrón-Quintana (2020), the welfare gain in Cho *et al.* (2015) can be thought of as a household-side version of the Oi-Hartmann-Abel effect, according to which firms can expand and contract production in response to positive and negative shocks, so as to take advantage of a mean-preserving spread to raise average output (see Oi, 1961; Hartmann, 1972; and Abel, 1983).

ability to reduce macroeconomic fluctuations may not be exhaustive, as also indicated by Jensen *et al.* (2018) and Lee *et al.* (2020). In fact, we show that the endogenous switching effect emerges as a relevant factor against which any potential stabilization benefits must be traded off. This appears particularly relevant for the emerging literature analyzing the effects of macroprudential policies, which has typically relied on models featuring some form of credit constraint on households and/or firms.

The paper is outlined as follows. Section 2 presents the model; Section 3 describes the solution method; Section 4 discusses the calibration; Section 5 reports the main result, as well as a number of exercises aimed at unveiling its key drivers; Section 6 concludes. Various technical details and supplementary material are reported in Appendices A–D.

2 The model

We consider a small open economy with free capital mobility. Time is discrete, $t = 1, 2, ..., \infty$. The economy is inhabited by representative households with utility

$$U = \mathbb{E}_1 \left[\sum_{t=1}^{\infty} \beta^{t-1} \left(\frac{1}{1-\gamma} c_t^{1-\gamma} + \frac{\nu}{1-\gamma_h} h_t^{1-\gamma_h} \right) \right], \tag{1}$$

where c_t is consumption of a perishable good, h_t is the stock of durables at the end of period t, with $\gamma > 0$, $\gamma_h > 0$ being coefficients of relative risk aversion, and $\nu > 0$ being a utility weight. $\mathbb{E}_t[.]$ denotes the rational expectations operator conditional on the period-t information set. Households borrow internationally at a fixed gross real interest rate $R > 1$. We assume that households are less patient than their foreign counterparts. Hence, the discount factor $0 < \beta < 1$ satisfies $\beta < R^{-1}$.

The flow budget constraint is

$$c_t + q_t (h_t - h_{t-1}) - d_t = y f(e_t) - R d_{t-1}, \qquad t = 1, 2, ..., \infty, \tag{2}$$

where q_t is the price of durables, d_{t-1} is one-period debt carried over from last period, y is time-invariant income, and f is a function of a log-normally distributed income shock, e_t. We assume $f(e_t) \equiv \exp\left(-\frac{1}{2}\sigma_e^2\right) \exp(e_t)$, where σ_e^2 is the unconditional variance of e_t, and where the first term in f cancels the positive average level effect on income that log-normality introduces. We assume that e_t is driven by an AR(1) process

$$e_{t+1} = \rho_e e_t + u_{t+1}^e, \qquad 0 < \rho_e < 1, \qquad u_{t+1}^e \sim \mathrm{N}\left(0, \sigma_{ue}^2\right). \tag{3}$$

Despite free capital mobility, households may be constrained in their amount of borrowing. We assume that debt must be partly collateralized by durables, à la Kiyotaki and Moore (1997). This stipulates that new borrowing, including interest, cannot exceed a time-varying fraction $s + s_t$ of the total expected value of durables:

$$d_t \leq (s + s_t) \frac{\mathbb{E}_t \left[q_{t+1}\right] h_t}{R}, \qquad t = 1, 2, ..., \infty, \tag{4}$$

where s is the average loan-to-value (LTV) ratio, and s_t captures a stochastic part of the LTV with unconditional variance σ_s^2. It can be shown that (4) will be binding in the steady state due to the assumption $\beta < 1/R$. This implies a determinate steady state. The feature is shared by a multitude of papers involving economies characterized by credit frictions, as well as within small-open economy applications on 'sudden stops'; see, e.g., Kiyotaki and Moore (1997), Iacoviello (2005), Jeanne and Korinek (2010), Bianchi (2011), Eggertsson and Krugman (2012), Liu *et al.* (2013), Liu and Wang (2014), Justiniano *et al.* (2015), Schmitt-Grohé and Uribe (2020), *inter alia.*[4]

The LTV shock evolves according to

$$s_{t+1} = \rho_s s_t + u_{t+1}^s, \qquad 0 < \rho_s < 1, \qquad u_{t+1}^s \sim \mathrm{N}\left(0, \sigma_{us}^2\right), \tag{5}$$

and following a large literature we interpret variations in s_t as shorthand for stochastic changes in the economy's financial conditions; see, e.g., Jermann and Quadrini (2012), Liu *et al.* (2013), Boz and Mendoza (2014), Bianchi and Mendoza (2018), and Jones *et al.* (2018).

Households maximize U subject to (2) and (4), taking as given $q_t > 0$ and the values of the states d_{t-1}, $h_{t-1} > 0$, e_t and s_t. The optimality conditions are

$$c_t^{-\gamma} = \Lambda_t, \tag{6}$$

[4] A body of research on 'sudden stops' follows Mendoza (2010), where a credit constraint does not bind in the steady state. He achieves a determinate steady state by adopting Epstein (1983) preferences, where discounting is a function of past consumption, and calibrates this function such that the credit constraint does not bind in the steady state.

$$\Lambda_t = \beta R \mathbb{E}_t \left[\Lambda_{t+1}\right] + \mu_t, \tag{7}$$

$$\Lambda_t q_t = \nu h_t^{-\gamma_h} + \beta \mathbb{E}_t \left[\Lambda_{t+1} q_{t+1}\right] + (s + s_t) \frac{\mathbb{E}_t \left[q_{t+1}\right]}{R} \mu_t, \tag{8}$$

where $\Lambda_t > 0$ and $\mu_t \geq 0$ are the multipliers associated with (2) and (4), respectively. We combine (6), (7) and (8) into the conventional Euler equations for optimal intertemporal consumption of perishable and durable goods, respectively:

$$c_t^{-\gamma} = \beta R \mathbb{E}_t \left[c_{t+1}^{-\gamma}\right] + \mu_t, \tag{9}$$

$$c_t^{-\gamma} q_t = \nu h_t^{-\gamma_h} + \beta \mathbb{E}_t \left[c_{t+1}^{-\gamma} q_{t+1}\right] + (s + s_t) \frac{\mathbb{E}_t \left[q_{t+1}\right]}{R} \mu_t. \tag{10}$$

3 Equilibrium and solution procedure

The market for durables is simplified by assuming that supply is constant, i.e.

$$h_t = h > 0, \qquad t = 0, 1, 2, \ \dots \ , \infty \,, \tag{11}$$

holds in all periods. Based on this, we can then state:

Definition 1 *An equilibrium is a set of functions d, c, q and μ that, conditional on d_{t-1} and $z_t \equiv [e_t, s_t]$, satisfy (2), (4), (9), (10). An equilibrium therefore satisfies*

$$c\left(d_{t-1}, z_t\right) + R d_{t-1} = y f\left(e_t\right) + d\left(d_{t-1}, z_t\right), \tag{12}$$

$$c\left(d_{t-1}, z_t\right)^{-\gamma} = \beta R \mathbb{E}_t \left[c\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)^{-\gamma}\right] + \mu\left(d_{t-1}, z_t\right), \tag{13}$$

$$\begin{aligned} c\left(d_{t-1}, z_t\right)^{-\gamma} q\left(d_{t-1}, z_t\right) &= \nu h^{-\gamma_h} + \beta \mathbb{E}_t \left[c\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)^{-\gamma} q\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)\right] \\ &\quad + (s + s_t) \frac{\mathbb{E}_t \left[q\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)\right]}{R} \mu\left(d_{t-1}, z_t\right), \end{aligned} \tag{14}$$

$$\mu\left(d_{t-1}, z_t\right) \left[d\left(d_{t-1}, z_t\right) - (s + s_t) \frac{\mathbb{E}_t \left[q\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)\right] h}{R}\right] = 0, \tag{15}$$

where (15) is the complementary slackness condition associated with (4) and $\mu\left(d_{t-1}, z_t\right) \geq 0$, and where the exogenous disturbances, z_t, evolve according to (3) and (5).

Note that the exogenous stochastic variables e_t and s_t enter the equilibrium conditions (12)–(15) so that, when considering different mean-preserving spreads of each shock (Rothschild and Stiglitz, 1970, 1971), we do not introduce arbitrary exogenous mean level effects. Exogenous level effects and their accompanying biases have long been acknowledged in the literature of uncertainty shocks in business cycles; see, e.g., Rankin (1994).

We solve the non-linear system (12)–(15) numerically. The state space spanned by d_{t-1} and z_t is discretized by 2,501 points for debt, and a five-state Markov chain for each of the shocks. Through Euler-equation iteration we obtain approximate policy functions $d_t = d\left(d_{t-1}, z_t\right)$, $c_t = c\left(d_{t-1}, z_t\right)$, $q_t = q\left(d_{t-1}, z_t\right)$ and $\mu_t = \mu\left(d_{t-1}, z_t\right)$. The recursive nature of the policy functions enables us to solve for the value function $V_t \equiv V\left(d_{t-1}, z_t\right) = \left[1/\left(1-\gamma\right)\right] c\left(d_{t-1}, z_t\right)^{1-\gamma} + \left[1/\left(1-\gamma^h\right)\right] h^{1-\gamma_h} + \beta \mathbb{E}_t\left[V\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)\right]$, which will be the basis for welfare analyses. The solution algorithm is detailed in Appendix B.

4 Calibration

Following Bianchi and Mendoza (2018), we calibrate the model using data for the OECD countries, while resorting to U.S. data in a few cases where data is not available for all OECD members. One period is interpreted as a quarter, such that $R = 1.01$ implies the commonly assumed 4% annual real interest rate, as is standard in small open economy models (e.g., Bianchi, 2011; or Schmitt-Grohé and Uribe, 2020). We calibrate the value of β such that—given the characteristics of the shocks, which we describe below—the model matches the average skewness of consumption growth across all OECD countries for the period 1980:Q1 to 2019:Q4, which is -0.9.[5] As discussed by Jordà et al. (2020), the negative skewness of consumption in the data plays a crucial role in the evaluation of the welfare effects of business cycles, and is therefore important to match. In our model, the value of β is a key determinant of the frequency with which the collateral constraint becomes non-binding, and therefore—this being the only source of asymmetry in the model—of the degree of skewness in consumption. The calibration results in a value of $\beta = 0.967$; a number closely in line with those used in many existing studies (e.g., Benigno *et al.*, 2013, Reyes-Heroles and Tenorio, 2020; and Jensen *et al.*, 2020). The average LTV ratio is $s = 0.8$, which is in line with the cross-country evidence reported by Calza *et al.* (2013) for a range of advanced economies, and with the values used in Bianchi and Mendoza (2018). Households' coefficients of relative risk aversion are set to $\gamma = \gamma_h = 2$; in line with microeconometric evidence (e.g., Attanasio

[5] We obtain data from the OECD World Economic Outlook database. For some OECD countries the available data sample is shorter, and we simply include all available quarters for each country.

Table 1. Baseline parameter values

Parameter	Description	Value
R	Gross real rate of interest	1.01
β	Discount factor	0.967
γ	CRRA, perishable consumption utility	2
γ_h	CRRA, durable consumption utility	2
ν	Utility weight, durable consumption	0.048
s	Average LTV ratio	0.8
y	Average income	1
h	Supply of durables	1
σ_e	Unconditional variance of the income shock	0.015
ρ_e	Autoregressive parameter of the income shock	0.908
σ_s	Unconditional variance of the financial shock	0.016
ρ_s	Autoregressive parameter of the financial shock	0.934

and Weber, 1995) and much of the existing literature (e.g., De Santis, 2007; Benigno *et al.*, 2013; and Sosa-Padilla, 2018). Both the steady-state income and the stock of durables are normalized to 1. The price of durables is then determined by the preference parameter ν. We calibrate this parameter to obtain an annualized ratio of household debt to output of 0.63, which is the median value for this ratio across the group of OECD countries; see IMF (2017). This implies setting $\nu = 0.048$.

The parameters related to the shock processes are calibrated as follows. The income shock is parameterized so that the income process in the model matches the average standard deviation and autocorrelation of the gross domestic product at business-cycle frequencies across all OECD countries for the period 1980:Q1 to 2019:Q4.[6] This yields $\sigma_e = 0.015$ and $\rho_e = 0.908$. As for the financial shock, we are not aware of quarterly data for assets and liabilities of the household sector across OECD countries for a sufficiently long period.[7] Instead, we focus on U.S. data and use the series for the LTV ratio of households constructed by Jensen *et al.* (2020) based on Flow of Funds data. We then set the parameters of the financial shock so that the movements in the LTV ratio in the model match the fluctuations in this series at business-cycle frequencies for the period 1980:Q1-2019:Q4. This requires setting $\sigma_s = 0.016$ and $\rho_s = 0.934$. All parameter values are summarized in Table 1.

[6] To focus on fluctuations at business-cycle frequencies, we apply a band-pass filter with bounds of 6 and 32 quarters, as is common in the literature.

[7] The OECD collects data for most member countries only since the late 1990's or even later. This dataset is therefore heavily influenced by the Global Financial Crisis of 2007-09 and would thus lead to larger financial shocks. As will become clear below, this would only strengthen our main findings.

5 The welfare effects of business fluctuations

To measure the welfare costs of business cycles, we follow Lucas (1987) and ask by what percentage the stochastic consumption path should be increased to obtain the same unconditional welfare as in an economy with no shocks. As shown in Appendix C, this number is given by

$$\lambda = 100 \left[\left(\frac{\mathbb{E}\left[\overline{V}\left(d_{t-1}\right)\right] - u^h}{\mathbb{E}\left[V\left(d_{t-1}, z_t\right)\right] - u^h} \right)^{\frac{1}{1-\gamma}} - 1 \right], \tag{16}$$

where $\overline{V}\left(d_{t-1}\right)$ denotes equilibrium welfare in an economy with no shocks, and where $u^h \equiv [1/\left(1-\beta\right)]\left[\nu/\left(1-\gamma_h\right)\right] h^{1-\gamma_h}$. Based on this metric, the unconditional cost of business cycles amounts to -0.21% of quarterly consumption in perpetuity, i.e., a net welfare gain. While not being large in absolute value, this is much larger than Lucas's (1987) original number(s), and it is the existence of a gain that is of most interest to our analysis.

It is insightful to condition the welfare measure on the stock of debt and the shock realizations. To this end, the following measure of *conditional* welfare loss of business cycles can be derived (see Appendix C):

$$\lambda^c\left(d_{t-1}, z_t\right) = 100 \left[\left(\frac{\overline{V}\left(d_{t-1}\right) - u^h}{\mathbb{E}_t V\left(d_{t-1}, z_t\right) - u^h} \right)^{\frac{1}{1-\gamma}} - 1 \right]. \tag{17}$$

The left panel of Figure 1 shows that, irrespective of the history of debt, when shocks take on their average values, then $\lambda^c < 0$. Hence, the presence of business cycles is welfare enhancing, particularly when initial debt is close to the deterministic steady state and, therefore, the economy is prone to switching to a regime in which the constraint does not bind.

Now, consider the central panel of Figure 1. Here we examine two opposite initial conditions. A 'bad' state, where both exogenous shocks are one standard deviation below their means, and a 'good' state, where both shocks are one standard deviation above. Notice that, irrespective of initial debt, the cost of business cycles is positive, conditional on a bad economic state. In fact, the magnitude of the cost can be conspicuous for an economy in high debt (0.5–2% of consumption). In the good state, instead, the opposite holds true: Irrespective of initial debt, we appreciate a business cycle gain, which increases over the support of d_{t-1} (1.1–1.7% of consumption).

In light of this asymmetry—and to ascertain the origins of the welfare gain of business fluctuations—it is important to quantify the chances that financial leverage is endogenously

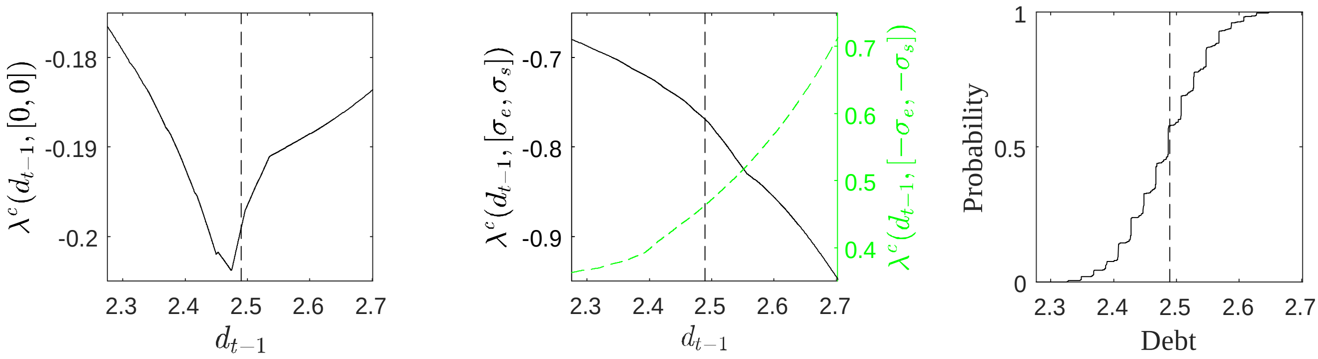

Figure 1: Conditional welfare losses and the stationary debt distribution. Left panel: Both shocks are initially at their means. Center panel: Both shocks are initially one s.d. higher (left axis, solid line), or one s.d. lower (right axis, green-dashed line) than their means. Right panel: Stationary cumulative distribution of debt. The vertical-dashed line denotes the deterministic steady-state debt. All parameters are at their baseline values.

driven to a 'costly' region of its support. In this respect, the right panel of Figure 1 reports the stationary cumulative distribution function of debt. The vertical line corresponds to mean debt in the deterministic economy, which amounts to 2.678.[8] Two insights are offered by this exercise: First, the distribution of debt is rather narrow around the deterministic level; second, the distribution is skewed to the left, so that 58% of the time d_t is lower than its counterpart in the deterministic case. Hence, it is relatively rare that debt may actually end up in the region where business cycles are very costly, as implied by both our computation of λ and the left panel of Figure 1. The next subsection will be devoted to understanding the key drivers of these results.

5.1 Why can business cycles be beneficial?

The kink in debt determination is crucial to the emergence of a business-cycle gain, as it allows for periods in which the constraint does not bind, through an *endogenous switching effect*. As a result of this, households can give vent to their inclination to smooth consumption, from time to time. In this respect, business cycles have an advantageous effect on welfare, as they induce occasional switches to an 'efficient' regime, with the endogenous switching effect dominating the *fluctuations effect* induced by the combination of uncertainty and consumers' prudence. We now turn to dissecting the drivers behind these forces, devising some comparative-statics exercises that emphasize the role of some key parameters.

[8] In the stochastic economy, instead, the mean is 2.676: This slightly lower figure arises from precautionary saving, and in itself would not necessarily give rise to cyclical gains. This difference also explains why λ^c reaches a trough slightly to the left of the deterministic steady-state debt level in the left panel of Figure 1.

Discount factor

It is important to start by observing the impact of households' degree of patience on unconditional welfare, as this has a tangible impact on their saving/consumption attitude. To this end, we examine welfare over a wide range of β's. The left panel of Figure 2 shows that, as consumers start with an implausibly high degree of impatience, the economy with uncertainty is welfare-dominated by the deterministic scenario. In this case, the credit constraint binds tightly, such that shocks never lead to the occurrence of episodes where agents are unconstrained; cf. the dashed-green line. Effectively, the endogenous switching effect is shut off when consumers are very impatient, so that only the traditional fluctuations effect is at work.

However, as β increases beyond a certain threshold—which lies well below the range of values typically considered in calibrations based on quarterly data—the cost of business cycles eventually translates into a steadily increasing gain. Thus, as households' *intertemporal saving motive* increases in their degree of patience, we observe an increasing frequency of episodes in which the credit constraint does not bind.

Risk aversion

The tension between financial tightness and the intensity of households' *precautionary saving motive* is central to our story. In this setting, precautionary saving arises from two sources. The first one is *prudence*. As discussed by Kimball (1990), the self-insurance is intimately linked to the coefficient of prudence, which under CRRA utility is given by $1+\gamma$. Therefore, the intensity of this saving motive increases in the curvature of households' utility function, as captured by the degree of risk aversion, γ.

In addition, households self-insure as they foresee the possibility that both current and future *borrowing constraints* bind, as it can be appreciated by iterating households' consumption Euler (9) forward:

$$c_t^{-\gamma} = \mu_t + \beta R \mathbb{E}_t \left[\mu_{t+1} + \beta R \mu_{t+2} + (\beta R)^2 \mu_{t+3} + ... \right]. \tag{18}$$

In a stochastic environment, the tightness of the financial constraint varies: Thus, at any point in time, even if $\mu_t = 0$, it must be that the expected term on the right side of (18) is positive, in light of $\mu_t \geq 0$, $\forall t$, and the fact that the borrowing constraint binds in the steady state, so that households cannot remain unconstrained indefinitely. Since households fear getting consecutive bad income realizations that would push them towards the borrowing

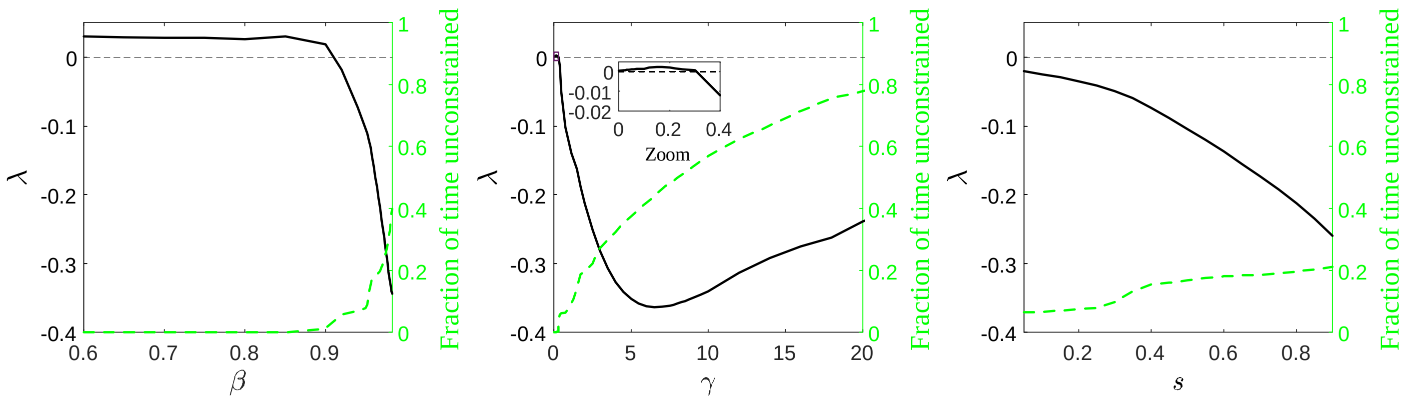

Figure 2: Welfare costs of business cycles for different values of the discount factor (left panel, the coefficient of relative risk aversion (center panel), and for different average LTV ratios (right panel). All other parameters are at their baseline values.

limit, they cut consumption. This allows them to increase their chances of facing a non-binding credit constraint in the future; i.e., the chance of switching into a different, and more favorable, economic regime. This second precautionary motive is crucial to the emergence of the endogenous switching effect.

With this in mind, we examine how risk aversion impacts on unconditional welfare in the central panel of Figure 2. Notably, when consumers are risk-neutral ($\gamma = 0$), the borrowing constraint binds and λ is virtually zero. When households become slightly risk-averse, fluctuations become costly, albeit very little (peaking at $\lambda \approx 0.002$ when $\gamma = 0.17$).[9] As γ increases further, λ dives into the negative territory. Compared with a frictionless economy where only prudence drives self-insurance, in the present setting higher risk aversion increases households' precautionary motive to the point that, eventually, consumption is taken to a regime compatible with the financial constraint being slack; and it does so more and more frequently. This entails a benefit to the consumer, who smooths consumption beyond what she is able to do when financially constrained, thus achieving a welfare gain from business cycles. As γ increases further, however, λ reverts its pattern, as households contract consumption further, and the endogenous switching effect gradually loses traction.[10] In essence, the gain from further reducing the risk of being constrained eventually gets counteracted—but not overcome—by the dislike of fluctuations *per se*.

[9] As illustrated in Figure D.1 in Appendix D, the welfare cost of fluctuations arising at low values of γ is driven by the presence of income shocks, as it arises also when we shut off financial shocks ($\sigma_s = 0$). In the absence of income shocks ($\sigma_e = 0$), however, business cycles are beneficial even as γ tends to zero.

[10] Notably, this happens well before the frequency of non-binding episodes approaches one, at which point the endogenous switching effect is exhausted.

To emphasize the role of precautionary saving in generating our key result, we run the model under perfect foresight, thus muting self-insurance. In this case, removing uncertainty implies that households' saving attitude is solely driven by a combination of intertemporal and smoothing motives, as captured by their discount factor and the curvature of their utility function. Each perfect foresight simulation lasts 400 periods, and the results are computed from 2,500 simulations, under the baseline calibration. Notably, $\lambda = 0.068$ in this scenario. This indicates that, being not faced with the need/option to save as a precautionary expedient to reduce the probability of hitting their borrowing limit in the future, households incur a cost from living in the stochastic economy.

LTV ratio

As depicted in the right panel of Figure 2, business fluctuations are beneficial even at extremely low LTV ratios. This occurs because the borrowing constraint may still become non-binding under these circumstances, e.g. due to credit-limit shocks.[11] As s rises, a gradual relaxation of the constraint occurs, resulting in increasingly frequent episodes of slackness, and thus an increasingly large welfare gain. In this case, the pattern of the gain over the s-support never reverts. These findings raise questions about the desirability of using caps on LTV ratios as a macroprudential policy tool, as done in several countries during the last decade (Cerutti *et al.*, 2017). In our setup, lowering the LTV ratio would be detrimental for welfare, even though such a policy reduces both the volatility and the negative skewness of consumption.

5.2 On the role of different shocks

To analyze the relative role of the shocks at play in the model, Figure 3 reports λ conditional on switching off either of them at the time. The left panel of the figure considers the case of no financial shocks ($\sigma_s = 0$): For an initial narrow range of their standard deviation, the income shocks hitting the economy are too small to make the financial constraint non-binding. In the absence of the endogenous switching effect, we observe that $\lambda > 0$ (albeit marginally). Increasing σ_e further reinforces the endogenous switching effect over the fluctuations effect. However, as income fluctuations become very large, self-insurance becomes conspicuous, eventually reducing steady-state consumption too much, and driving λ into the 'costly' region.

[11] In the absence of financial shocks, however, business cycles lead to a welfare loss at extremely low values of s, as shown in Figure D.2 in Appendix D. In that case, episodes of non-binding constraints are very infrequent.

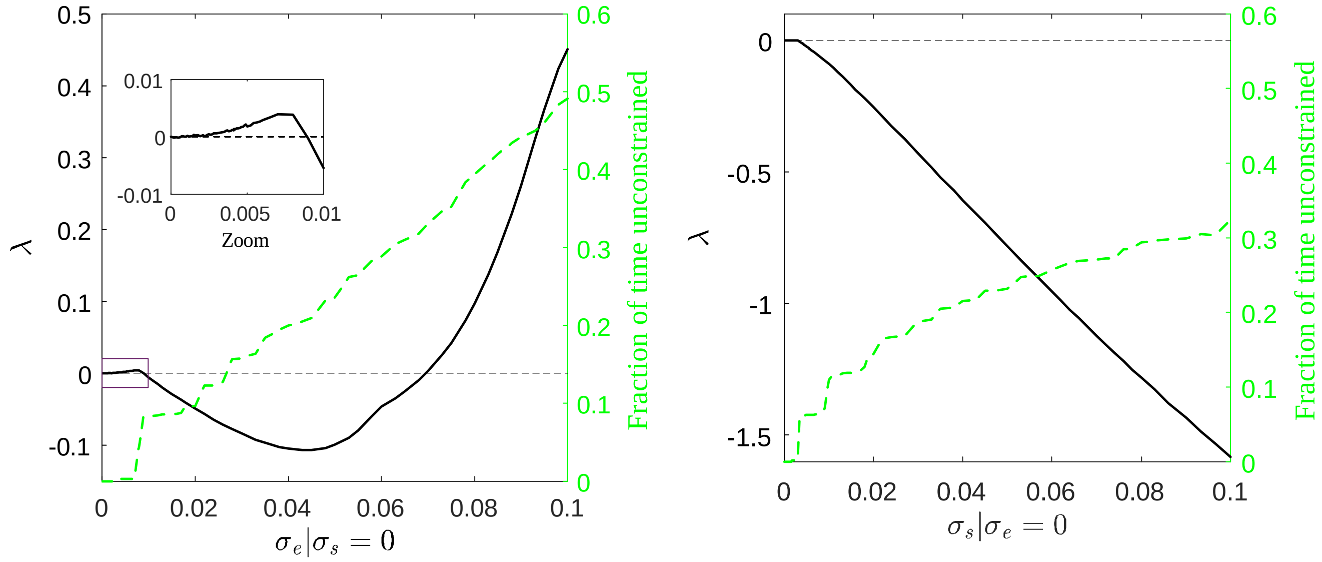

Figure 3: Welfare costs of business cycles for different standard deviations of a given shock, conditional the other shock being switched off. All the other parameters are at their baseline values.

By contrast, a rise in σ_s—absent income shocks ($\sigma_e = 0$)—translates into an increasing gain from business fluctuations (see the right panel of Figure 3).

The different role played by the two types of shocks is reminiscent of the result of Cho *et al.* (2015), who show that introducing multiplicative shocks implies that fluctuating economies may enjoy higher welfare, as compared with their no-shock counterparts. A *mean effect* of uncertainty is at work in their case—along with the usual fluctuations effect—as compared with situations in which shocks enter additively: Multiplicative shocks have the potential to raise the mean output and/or consumption, allowing consumers to take advantage of uncertainty by working harder and investing more during expansionary periods. By contrast, when uncertainty enters the economy additively, it has no beneficial effect on the choices that can be adjusted to it. This is the case for our income shock, which enters the budget constraint additively. On the other hand, the LTV shock affects the Euler equation for durables, (10), so that consumers can take advantage of periods of relatively lax financial conditions to smooth consumption. It is important to stress that this mean effect shall not be confused with the endogenous switching effect, but rather is complementary to it. This can be inferred from the fact that we still observe substantial welfare gains when the financial (and multiplicative) shock is switched off. In fact, as indicated by the left panel of Figure 3, abstracting from the mean effect implies that the endogenous switching effect channeled through the (additive) income shock alone still accounts for about 12% of the gain from business fluctuations at the

Table 2. Results with a fixed credit limit

Combination of shocks	Value of the welfare loss (λ)	Fraction of time unconstrained
Baseline (σ_e= 0.015, σ_s= 0.016)	−0.016	0.063
No income shocks (σ_e= 0)	−0.037	0.063
Large income shocks (σ_e= 0.03)	0.054	0.067
No credit-limit shocks (σ_s= 0)	0.022	0.000
Large credit-limit shocks (σ_s= 0.03)	−0.087	0.114

Notes: The table reports the unconditional welfare loss (λ) and the fraction of time households spend being financially unconstrained under fixed credit limits and various shock configurations.

baseline calibration (i.e., 0.025/0.21).

5.2.1 Fixed credit limits

The experiments featuring one type of shock at the time are very informative about the role of the financial constraints in generating gains from business fluctuations, depending on the shocks they typically amplify. To dig deeper on this, we consider the special case of an exogenous credit limit (i.e., where the collateral is always pledged at the steady-state value, such that (4) is replaced by $d_t \leq (s + s_t)\frac{qh}{R}$). In Table 2, we report the results from this model under various shock configurations. Notably, fluctuations are welfare improving under our baseline calibration, but the gain (−0.016% of consumption) is much smaller than in our baseline model with an endogenous credit limit (−0.21%), highlighting the role of the financial accelerator of Kiyotaki and Moore (1997). Furthermore, the results in the table reflect that, once again, it is the credit-limit shock that crucially contributes to the emergence of gains from business cycles. When we shut off this shock, so that the borrowing limit is completely fixed, business cycles become costly. This is consistent with the findings of İmrohoroğlu (1989) in a model with an Aiyagari (1994)-style constant borrowing limit, which by construction does not embody the financial accelerator. In general, we observe that fluctuations are more beneficial or less costly when credit-limit shocks are relatively large and income shocks relatively small, and vice versa. In the absence of credit-limit disturbances, very large income shocks are required to make households unconstrained. This implies that the endogenous switching effect only comes into play when the fluctuations effect is already very strong.

Incidentally, the experiment with fixed credit limits also sheds light on the role of pecuniary externalities—i.e., the fact that households' optimizing behavior affects the collateral price (see, e.g., Lorenzoni, 2008 and Bianchi, 2011)—for the emergence of business-cycle gains when collateral constraints determine a kink in households' policy functions. While increasing the chances that the financial constraint becomes non-binding in the face of an expansionary shock, thus expanding households' borrowing capacity, Table 2 shows this element is not key to the emergence of welfare gains.

6 Concluding remarks

This paper considers the welfare cost of business cycles in a credit economy with collateralized debt. Welfare tends to be higher in the economy with aggregate fluctuations, as compared with the benchmark model without uncertainty, as shocks facilitate endogenous switching from 'bad' to 'good' economic regimes. This benefit may outweigh the conventional losses from business fluctuations.

This basic insight has key implications for stabilization policies that affect the frequency at which collateral constraints may become slack, allowing agents to behave at least temporarily as standard consumption smoothers. In this respect, in Jensen *et al.* (2018) we establish a macroeconomic volatility trade-off that arises from collateral constraints not binding at all points in time: On one hand, a reduction of credit may dampen the asset-price sensitivity of those borrowers who remain credit constrained before and after the intervention; on the other hand, lower credit limits increase the frequency at which credit constraints bind, thus augmenting borrowers' sensitivity to fluctuations in credit availability. In a similar fashion, though in an environment with idiosyncratic risk and incomplete markets, Lee *et al.* (2020) show that a stricter regulation of leverage in the banking sector inhibits households' ability to smooth consumption in response to idiosyncratic risk. Thus, while this type of restrictions might act as stabilizers at the macroeconomic level, they do not necessarily stabilize at the microeconomic level, potentially resulting in substantial welfare costs. In sum, both contributions indicate the importance of assessing agents' position with respect to kinks in their policy functions, as well as the quantitative importance of facing non-binding constraints, as compared with the traditional welfare-reducing effects of uncertainty. We see this aspect as of crucial importance in the design of sound macroprudential policies.

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Appendices

A Deterministic steady state

In the absence of any shocks, we have from (13) that

$$\begin{aligned} \mu &= (c)^{-\gamma} - \beta R \mathbb{E}\left[(c)^{-\gamma}\right], \\ &= (c)^{-\gamma}(1-\beta R) > 0, \end{aligned} \tag{A.1}$$

where undated variables denote deterministic steady-state values, and where the inequality sign follows from $\beta < 1/R$. From (15) we therefore get

$$d = s\frac{qh}{R}. \tag{A.2}$$

From (12) and (A.2) we get

$$c + sqh\left(1 - R^{-1}\right) = y. \tag{A.3}$$

By (14), we get

$$\nu\,(h)^{-\gamma_h} = (c)^{-\gamma}\,q - \beta q c^{-\gamma} - s\frac{q}{R}\mu,$$

which by (A.1) becomes

$$\begin{aligned} \nu\,(h)^{-\gamma_h} &= (c)^{-\gamma}\,q - \beta q\,(c)^{-\gamma} - s\frac{q}{R}\,(c)^{-\gamma}\,(1-\beta R), \\ &= (c)^{-\gamma}\,q\left[1 - \beta - s\frac{1}{R}\,(1-\beta R)\right]. \end{aligned} \tag{A.4}$$

Equations (A.3) and (A.4) provide the unique solutions for c and q. Conditional on these, closed-form solutions for μ and d follow from (A.1) and (A.2), respectively.

B Solution algorithm

We solve the model numerically through Euler-equation iteration on the policy functions. Our problem is non-standard because the credit constraint introduces a state-dependent discontinuity. As argued by Rendahl (2015), searches for solutions can in such cases be divergent, cyclical, or even non-convergent. We therefore follow Judd (1988) and introduce 'dampening' parameters in the updating of policy functions. This implies that in any update of a policy function, only a fraction of the new function will replace the old. This fosters convergence.

We first discretize the state variables d_{t-1}, e_t, and s_t such that $d_{t-1} \in \mathbf{d}_{t-1} \equiv [d_{\min}, \ldots, d_{\max}]^\intercal$, $e_t \in \mathbf{e}_t \equiv [e_{\min}, \ldots, e_{\max}]^\intercal$, $s_t \in \mathbf{s}_t \equiv [s_{\min}, \ldots, s_{\max}]^\intercal$. In the construction of $\mathbf{d}_{t-1}$ we allow for a relatively finer grid around the deterministic steady state to foster precision. In the construction of state vectors we secure that the model does not imply starvation for high initial debt combined with sufficiently adverse shocks. The discretization of the shocks uses Rouwenhorst's (1995) method of approximating AR(1) processes by Markov chains with transition matrices, $\mathbf{P}_\mathbf{e}$ and $\mathbf{P}_\mathbf{s}$, respectively. We thereby follow Kopecky and Suen (2010), who find that this method best approximates very persistent processes compared with other methods. To simplify notation and computations, we create a column vector of all shock combinations, $\mathbf{z}_t \equiv \text{vec}\left(\mathbf{s}_t\mathbf{e}_t^\intercal\right)$. The associated transition matrix for $\mathbf{z}_t$ is then given by $\mathbf{\Pi} \equiv \mathbf{P}_\mathbf{e} \otimes \mathbf{P}_\mathbf{s}$, where $\otimes$ is the Kronecker product. We use 2,501 debt states and five states for each shock.

In the solution procedure we form a matrix of all state combinations, $\mathbf{d}_{t-1}\mathbf{z}_t^\intercal$, and seek solutions for policy functions yielding matrices $c\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, $q\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, $d\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, and $\mu\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, which satisfy the equilibrium conditions. Note that in any state, we have either $\mu_t = 0$ or $\mu_t > 0$. Call these two cases the *unconstrained* and *constrained* regime, respectively. In each iteration we solve the model in two blocks—one for each regime. This exploits the different structure of the solution in either regime.[12] Subsequently, the policy matrices are appropriately merged before proceeding with the next iteration. The algorithm is as follows:

1. Make initial guesses $c^0\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$ and $q^0\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$.

2. Use (12) to obtain $d\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = c^i\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) + R\widetilde{\mathbf{d}}_{t-1} - yf\left(\widetilde{\mathbf{e}}_t\right)$, where $\widetilde{\mathbf{d}}_{t-1}$ is a matrix of repeated columns of $\mathbf{d}_{t-1}$ conformable with $\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}$. The income shocks are used to construct the matrix $f\left(\widetilde{\mathbf{e}}_t\right)$, which has identical row vectors of the possible income-shock values, conformable with $\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}$.

3. Use $\mathbf{d}_t = d\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$ to compute $\mathbf{c}_{t+1} = \widehat{c}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)$ and $\mathbf{q}_{t+1} = \widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)$ through column-wise interpolation on $\mathbf{d}_{t-1}$ and $c^i\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$ and $q^i\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, respectively.

4. Derive policy functions in the *unconstrained* regime:

   (a) By definition, $\mu^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = 0$.

   (b) From (13),
   $$c^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = \left\{\beta R\left[\widehat{c}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)^{-\gamma}\mathbf{\Pi}^{\mathsf{T}}\right]\right\}^{-1/\gamma}.$$

   (c) From (14),
   $$\begin{aligned} q^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) &= c^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)^{\gamma} \\ &\circ\left\{\nu\left(h\right)^{-\gamma_h} + \beta\left[\left(\widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)\circ\widehat{c}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)^{-\gamma}\right)\mathbf{\Pi}^{\mathsf{T}}\right]\right\}, \end{aligned}$$
   where $\circ$ denotes element-by-element multiplication.

   (d) By (12), find $d^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = c^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) + R\widetilde{\mathbf{d}}_{t-1} - yf\left(\widetilde{\mathbf{e}}_t\right)$.

5. Derive policy functions in the *constrained* regime:

   (a) Let the matrix $\widetilde{\mathbf{s}}_t$ contain identical row vectors of the possible LTV-shock values, conformable with $\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}$. In each column of $\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}$ identify the states where
   $$d^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) > \left[\left(s+\widetilde{\mathbf{s}}_t\right)/R\right]\circ\left[\widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)\mathbf{\Pi}^{\mathsf{T}}\right]h,$$
   as these violate (4) and therefore characterize the constrained regime. For any matrix $\mathbf{X}_t$, denote by $\left[\mathbf{X}_t\right]^j$ the j^{th} column of $\mathbf{X}_t$ only consisting of such identified states.

   (b) From (4), when the constraint binds,
   $$\left[d^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^j = \left(\left[s+\widetilde{\mathbf{s}}_t\right]^j/R\right)\circ\left[\widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)\mathbf{\Pi}^{\mathsf{T}}\right]^j h, \quad \text{all } j.$$

   (c) From (15),
   $$\left[c^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^j = y\left[f\left(\widetilde{\mathbf{e}}_t\right)\right]^j - R\left[\widetilde{\mathbf{d}}_{t-1}\right]^j + \left[d^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^j, \quad \text{all } j.$$

[12] This approach is in accordance with the one suggested by Jeanne and Korinek (2010).

(d) From (13),

$$\left[\mu^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^j = \left(\left[c^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^j\right)^{-\gamma} - \beta R\left[\widehat{c}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)^{-\gamma}\mathbf{\Pi}^{\mathsf{T}}\right]^j, \quad \text{all } j.$$

(e) From (14),

$$\begin{aligned}\left[q^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^j \;=\; & \left(\left[c^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^j\right)^{\gamma} \\ & \circ\Bigg\{\nu\left(h\right)^{-\gamma_h} + \beta\left[\left(\widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)\circ\widehat{c}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)^{-\gamma}\right)\mathbf{\Pi}^{\mathsf{T}}\right]^j \\ & + \left(\left[s+\widetilde{\mathbf{s}}_t\right]^j/R\right)\circ\left[\widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)^{-\gamma}\mathbf{\Pi}^{\mathsf{T}}\right]^j\circ\left[\mu^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^j\Bigg\}, \quad \text{all } j.\end{aligned}$$

6. An updated set of policy functions $c^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, $q^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, and the associated $d\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$ and $\mu\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, are built from the respective matrices found in the unconstrained and constrained regimes. Specifically, in the policy matrices derived for the unconstrained regime, replace the values with the ones found in the constrained regime for the states identified in Step 5a.

7. If

$$\left\|\text{vec}\left[c^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right] - \text{vec}\left[c^{i}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]\right\|_{\infty} < \varepsilon,$$

and

$$\left\|\text{vec}\left[q^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right] - \text{vec}\left[q^{i}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]\right\|_{\infty} < \varepsilon,$$

where ε is some tolerance criterion, then stop (we use $\varepsilon = 10^{-8}$). Otherwise, update according to $c^{i+2}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = \omega_c c^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) + \left(1-\omega_c\right)c^{i}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$ and $q^{i+2}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = \omega_q q^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) + \left(1-\omega_q\right)q^{i}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, where $0 < \omega_c,\ \omega_q < 1$ are dampening parameters, and go to 2.

Subsequently, the value function is computed. Start with a guess $V^0\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$. Then proceed as follows:

1. Use $d\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$ to obtain $V_{t+1} = \widehat{V}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)$ through column-wise interpolation on $\mathbf{d}_{t-1}$ and $V^i\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$.

2. Compute

$$V^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = \frac{1}{1-\gamma}\left[c\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^{1-\gamma} + \frac{\nu}{1-\gamma_h}\left(h\right)^{1-\gamma_h} + \beta\widehat{V}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)\mathbf{\Pi}^{\mathsf{T}}.$$

3. If

$$\left\|\text{vec}\left[V^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right] - \text{vec}\left[V^{i}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]\right\|_{\infty} < \varepsilon,$$

where ε is the tolerance criterion, then stop. Otherwise, set $V^{i+2}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = \omega_V V^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) + \left(1-\omega_V\right)V^{i}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, where $0 < \omega_V < 1$ is a dampening parameter, and go to 1.

C Derivation of the costs of business cycles

We want to find the value of λ that secures $\mathbb{E}\left[V\left(d_{t-1}, z_t\right)\right] = \mathbb{E}\left[\overline{V}\left(d_{t-1}\right)\right]$; i.e., indifference between the stochastic and non-stochastic economies. Using the definitions of the value functions together with the definition of λ as the percentage increase in the consumption path in the stochastic economy to secure indifference, λ satisfies

$$\begin{aligned} & \mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\left(\frac{1}{1-\gamma}\left[\left(1+\lambda/100\right)c\left(d_{t-1},z_t\right)\right]^{1-\gamma}+\frac{\nu}{1-\gamma_h}h^{1-\gamma_h}\right)\right] \\ = & \mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\left(\frac{1}{1-\gamma}\left[\overline{c}\left(d_{t-1}\right)\right]^{1-\gamma}+\frac{\nu}{1-\gamma_h}h^{1-\gamma_h}\right)\right], \end{aligned} \tag{C.1}$$

where $\overline{c}\left(d_{t-1}\right)$ is the policy function for consumption under certainty. From (C.1) we readily obtain

$$\left(1+\lambda/100\right)^{1-\gamma}\mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\frac{1}{1-\gamma}\left[c\left(d_{t-1},z_t\right)\right]^{1-\gamma}\right]=\mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\frac{1}{1-\gamma}\left[\overline{c}\left(d_{t-1}\right)\right]^{1-\gamma}\right],$$

and therefore

$$\begin{aligned} \left(1+\lambda/100\right)^{1-\gamma} & = \frac{\mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\frac{1}{1-\gamma}\left[\overline{c}\left(d_{t-1}\right)\right]^{1-\gamma}\right]}{\mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\frac{1}{1-\gamma}\left[c\left(d_{t-1},z_t\right)\right]^{1-\gamma}\right]}, \\ & = \frac{\mathbb{E}\left[\overline{V}\left(d_{t-1}\right)\right]-u^h}{\mathbb{E}\left[V\left(d_{t-1},z_t\right)\right]-u^h}, \end{aligned} \tag{C.2}$$

where the second line in (C.2) follows from the definitions of the value functions and u^h. From (C.2), we immediately recover the unconditional welfare measure, as desired.

As for the conditional welfare measure, $\lambda^c\left(d_{t-1},z_t\right)$, this satisfies

$$\begin{aligned} & \mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\left(\frac{1}{1-\gamma}\left[\left(1+\lambda^c\left(d_{t-1},z_t\right)/100\right)c\left(d_{s-1},z_s\right)\right]^{1-\gamma}+\frac{\nu}{1-\gamma_h}h^{1-\gamma_h}\right)\right] \\ = & \mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\left(\frac{1}{1-\gamma}\left[\overline{c}\left(d_{s-1}\right)\right]^{1-\gamma}+\frac{\nu}{1-\gamma_h}h^{1-\gamma_h}\right)\right]. \end{aligned} \tag{C.3}$$

Similar manipulations as used above in the case of (C.1), readily yield (17).

D Additional figures

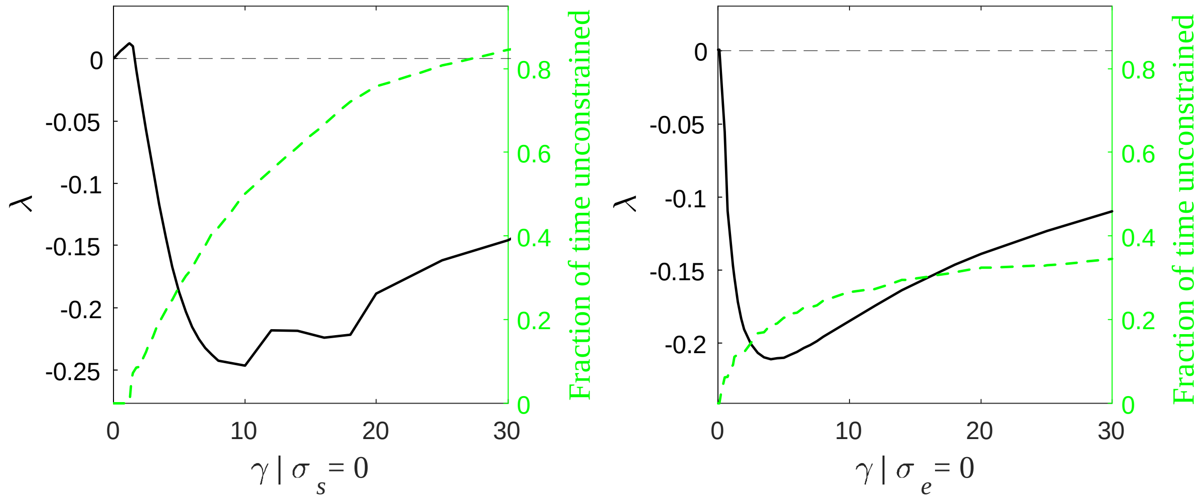

Figure D.1: Welfare costs of business cycles for different values of relative risk aversion, in the absence of LTV shocks (left panel) and income shocks (right panel). All the other parameters are at their baseline values.

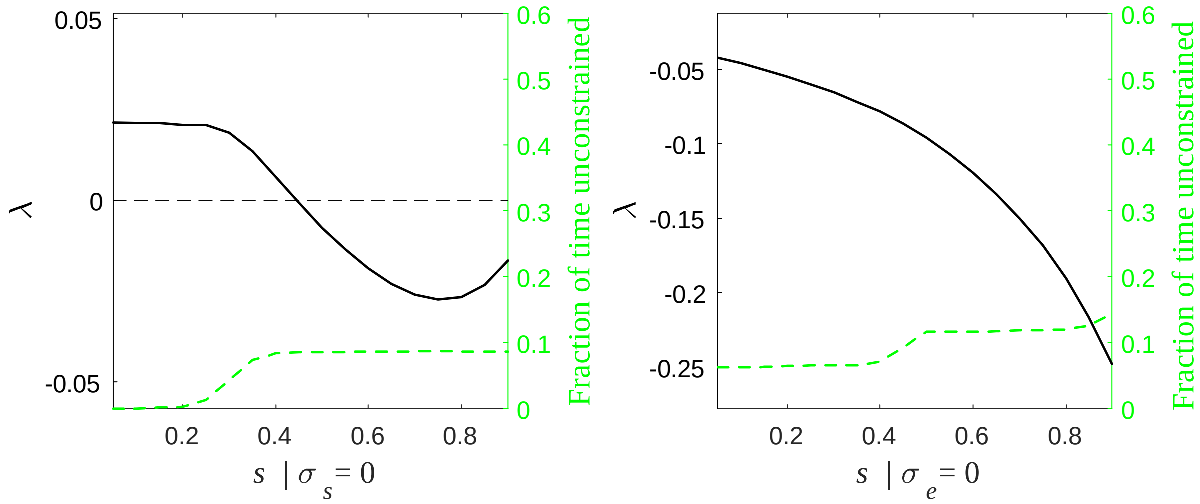

Figure D.2: Welfare costs of business cycles for different average LTV ratios in the absence of LTV shocks (left panel) and income shocks (right panel). All the other parameters are at their baseline values.