Kinks and Gains from Credit Cycles*

Henrik Jensen[†]
University of Copenhagen and CEPR

Søren Hove Ravn[‡]
University of Copenhagen

Emiliano Santoro[§]
University of Copenhagen

June 2019

Abstract

Credit-market imperfections are at the centre stage of several theories of business fluctuations. Since a lot of research seeks to address the welfare consequences of stabilization policies, we revisit the fundamental question of quantifying the cost of business cycles in a model where household borrowing is subject to a collateral constraint. Business cycles occasionally change the credit-market conditions, making households temporarily unconstrained and better off. This effect can dominate the conventional losses from uncertainty, thus making fluctuations welfare-dominate certainty.

Keywords: Cost of business cycles, collateral constraints, precautionary saving.

JEL codes: E20, E32, E66.

*We thank Gianluca Benigno, Jeppe Druedahl, Ivan Petrella, Pontus Rendahl and Kjetil Storesletten for helpful discussions. The usual disclaimer applies.

[†]University of Copenhagen and CEPR. Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Bld. 26, 1353 Copenhagen, Denmark. *E-mail*: Henrik.Jensen@econ.ku.dk.

[‡]University of Copenhagen. Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Bld. 26, 1353 Copenhagen, Denmark. *E-mail*: Soren.Hove.Ravn@econ.ku.dk.

[§]University of Copenhagen. Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Bld. 26, 1353 Copenhagen, Denmark. *E-mail*: Emiliano.Santoro@econ.ku.dk.

1 Introduction

Much macroeconomic research is devoted to understanding business cycles. This is a natural consequence of the booms and downturns in economic activity, which have been observed throughout the world since the invention of formal national accounts. Closely related to this line of research are analyses of which kind of economic stabilization policies could possibly attenuate cyclical movements in economic activity. In order to advocate a role for such policies it does not seem unreasonable to identify relevant welfare costs entailed by the business fluctuations one suggests to dampen (Lucas, 1987). The usual qualitative argument for business cycle costs takes as a starting point a concave and continuous welfare function of some state of the economy. Thus, one compares the outcome from the deterministic case with its counterpart in the stochastic case, where the state fluctuates around the deterministic value. By Jensen's inequality, the former outcome is preferred.

While Lucas's seminal contribution did not question this qualitative argument, it strongly questioned its quantitative relevance. Using a conventional CRRA utility function, he computed the welfare loss of cyclical variations, defined in terms of how much consumption is needed to compensate an economic agent for the presence of business fluctuations. For parameter values that are conventional in the macroeconomic literature, he found the losses as low as 0.008% of consumption. This negligible size has since been contested by a vast literature identifying higher welfare costs of fluctuations in extensions of Lucas's simple framework.[1] While some significant increases have been recovered, consensus still seems to be that the cost of business cycle is small.[2]

Over the last decade macroeconomic research has revived the interest in modelling credit-market imperfections. Likewise, the demand for macroprudential policies capable of dampening fluctuations arising from, or magnified by, credit markets is high on the policy agenda. We accordingly revisit the task of quantifying the cost of business cycles in a simple small-open economy model where borrowing is subject to a collateral constraint in the vein of Kiyotaki and Moore (1997). This type of constraint has become a common ingredient in dynamic general equilibrium models employed to highlight the role of credit-market imperfections, in

[1] These include, but are not limited to, models with incomplete financial markets (İmrohoroğlu, 1989; Krusell and Smith, 1999; Krusell *et al.*, 2009; Storesletten *et al.*, 2001), imperfect competition (Galí *et al.*, 2007), detailed time-series modelling of consumption (Reis, 2007; De Santis, 2007), non-expected utility functions (Obstfeld, 1994), asset prices (Alvarez and Jermann, 2004), endogenous growth (Barlevy, 2004), and disaster risk (Barro, 2006).

[2] It has been established, however, that welfare losses conditional on experiencing particularly bad episodes can be high; see, e.g., Galí *et al.* (2007).

both positive and normative analyses. It stipulates that an individual cannot borrow more than a fraction of the value of some collateral, which is typically represented by a durable good (e.g., housing). This gives rise to the well-known financial accelerator effect, according to which shocks to the economy are amplified through asset-price movements. The constraint introduces a discontinuity that plays a central role: It is either binding—the so-called 'constrained regime'—or not—the 'unconstrained regime'. Policy functions will therefore feature a *kink* at the point where the model switches from one regime to the other.

In this setting, we find that business cycles may be *beneficial* for welfare, with the benefit being one order of magnitude larger than Lucas's number: Fluctuations raise unconditional welfare by around 0.25% of consumption. As agents in the domestic economy are more impatient than international lenders—and therefore prone to borrowing—the steady state of the model is characterized by a binding credit constraint.[3] This source of inefficiency restricts consumption below the level attainable if agents were able to act as standard consumption smoothers—thus reducing their lifetime utility. Gains emerge as, being subject to fluctuations generated by stochastic disturbances, the economy displays episodes of non-binding credit constraints that temporarily alleviate the key source of inefficiency in the economy.

The emergence of a cost of business fluctuations in the existing literature typically depends on the interplay between uncertainty and consumers' preference to smooth consumption over time, as embodied by a concave utility function. In other words, in the presence of risk aversion, agents would typically prefer a stable consumption path, as compared with one that fluctuates around the same mean. As a result, some compensation would be necessary to make consumers indifferent between the two consumption paths. This traditional mechanism—which will be referred to as the *fluctuations effect*—is at play in our model of dynamic consumption-saving decisions.

We use a conventional utility function exhibiting prudence (Kimball, 1990), where uncertainty about income and financial conditions leads to precautionary saving. The introduction of a financial constraint induces an additional precautionary motive, as consumers try to reduce the risk of being financially constrained. The arrival of shocks, combined with households' decisions, endogenously determine whether the credit constraint binds or not. The resulting kink in debt determination is crucial, as it facilitates temporary switches to a regime in which the constraint does not bind, allowing households to smooth consumption

[3] This is customary in the long-standing tradition of dynamic stochastic general equilibrium models with credit constraints, ever since Kiyotaki and Moore (1997).

efficiently, from time to time. In this respect, shocks also have an advantageous effect on welfare, as they induce occasional switches to an 'efficient' regime. In the remainder, we refer to this effect as the *endogenous switching effect*. For realistic parameter values, we find this effect to be stronger than the fluctuations effect, thus paving the way for business cycles to entail a gain.

We have already pointed to the existence of a large literature involved in the assessment of the cost of business fluctuations. We have also stressed that most of these contributions have typically been seeking for empirical or technical features capable of inflating the welfare cost, while the hypothesis that business fluctuations could instead be welfare enhancing has rarely been supported, or even advanced. In this respect, closely related to our paper is Cho *et al.* (2015), who find that gains from business cycles may arise in a conventional real business cycle model where labor supply is a convex function of the shocks. The presence of multiplicative shocks is crucial for this result: Such shocks have the potential to raise the mean level of output and/or consumption, allowing agents to take advantage of uncertainty by working harder and investing more during expansionary periods. By contrast, when uncertainty enters the economy additively, it has no beneficial effect on the choices that can be adjusted to it. While a *mean effect* of uncertainty is at play also in our setup, our endogenous switching effect operates independently and produces gains from business cycles, even when only additive shocks are at play.

Our findings have important implications for the assessment and the conduct of economic stabilization policies. In an environment where the credit constraints faced by households are not binding, from time to time, evaluating the desirability of such policies solely based on their ability to reduce macroeconomic fluctuations may not be appropriate. In this type of context, the endogenous switching effect may emerge as a relevant factor against which any potential stabilization benefits must be traded off. This appears particularly relevant for the emerging literature analyzing the effects of macroprudential policies, which has typically relied on models featuring some form of credit constraint on households and/or firms.

The paper is outlined as follows. Section 2 presents the model; Section 3 describes the solution method; Section 4 reports the main results, as well as a number of robustness exercises; Section 5 concludes. Various technical details and supplementary material are reported in Appendices A–D.

2 The model

We consider a small open economy with free capital mobility. Time is discrete, $t = 1, 2, ..., \infty$. The economy is inhabited by representative households with utility

$$U = \mathbb{E}_1 \left[\sum_{t=1}^{\infty} \beta^{t-1} \left(\frac{1}{1-\gamma} c_t^{1-\gamma} + \frac{\nu}{1-\gamma_h} h_t^{1-\gamma_h} \right) \right], \tag{1}$$

where c_t is consumption of a perishable good, h_t is the stock of durables at the end of period t, with $\gamma > 0$, $\gamma_h > 0$ being coefficients of relative risk aversion, and $\nu > 0$ being a utility weight. $\mathbb{E}_t [.]$ denotes the rational expectations operator conditional on the period-t information set. Households borrow internationally at a fixed gross real interest rate $R > 1$. We assume that households are less patient than their foreign counterparts. Hence, the discount factor $0 < \beta < 1$ satisfies $\beta < R^{-1}$.

The flow budget constraint is

$$c_t + q_t (h_t - h_{t-1}) - d_t = y f(e_t) - R d_{t-1}, \qquad t = 1, 2, ..., \infty, \tag{2}$$

where q_t is the price of durables, d_{t-1} is one-period debt carried over from last period, y is time-invariant income, and f is a function of a log-normally distributed income shock, e_t. We assume $f(e_t) \equiv \exp\left(-\frac{1}{2}\sigma_e^2\right) \exp(e_t)$, where σ_e^2 is the unconditional variance of e_t, and where the first term in f cancels the positive average level effect on income that log-normality introduces. We assume that e_t is driven by an AR(1) process

$$e_{t+1} = \rho_e e_t + u_{t+1}^e, \qquad 0 < \rho_e < 1, \qquad u_{t+1}^e \sim \mathrm{N}\left(0, \sigma_{ue}^2\right). \tag{3}$$

Despite free capital mobility, households may be constrained in their amount of borrowing. We assume that debt must be partly collateralized by durables, à la Kiyotaki and Moore (1997). This stipulates that new borrowing, including interest, cannot exceed a time-varying fraction $s + s_t$ of the total expected value of durables:

$$d_t \leq (s + s_t) \frac{\mathbb{E}_t [q_{t+1}] h_t}{R}, \qquad t = 1, 2, ..., \infty, \tag{4}$$

where s is the average loan-to-value (LTV) ratio, and s_t captures a stochastic part of the

LTV with unconditional variance σ_s^2.[4] It can be shown that (4) will be binding in the steady state due to the assumption $\beta < 1/R$. This implies a determinate steady state. The feature is shared by a multitude of papers involving economies characterized by credit frictions, as well as within small-open economy applications on 'sudden stops'; see, e.g., Kiyotaki and Moore (1997), Iacoviello (2005), Jeanne and Korinek (2010), Bianchi (2011), Eggertsson and Krugman (2012), Liu *et al.* (2013), Liu and Wang (2014), Justiniano *et al.* (2015), Schmitt-Grohé and Uribe (2016), *inter alia*.[5]

The LTV shock evolves according to

$$s_{t+1} = \rho_s s_t + u_{t+1}^s, \qquad 0 < \rho_s < 1, \qquad u_{t+1}^s \sim \mathrm{N}\left(0, \sigma_{us}^2\right), \tag{5}$$

and following a large literature we interpret variations in s_t as shorthand for stochastic changes in the economy's financial conditions; see, e.g., Jermann and Quadrini (2012), Liu *et al.* (2013), Boz and Mendoza (2014), Bianchi and Mendoza (2018), and Jones *et al.* (2018).

Households maximize U subject to (2) and (4), taking as given $q_t > 0$ and the values of the states d_{t-1}, $h_{t-1} > 0$, e_t and s_t. The optimality conditions are

$$c_t^{-\gamma} = \Lambda_t, \tag{6}$$

$$\Lambda_t = \beta R \mathbb{E}_t \left[\Lambda_{t+1}\right] + \mu_t, \tag{7}$$

$$\Lambda_t q_t = \nu h_t^{-\gamma_h} + \beta \mathbb{E}_t \left[\Lambda_{t+1} q_{t+1}\right] + (s + s_t) \frac{\mathbb{E}_t \left[q_{t+1}\right]}{R} \mu_t, \tag{8}$$

where $\Lambda_t > 0$ and $\mu_t \geq 0$ are the multipliers associated with (2) and (4), respectively. We combine (6), (7) and (8) into the conventional Euler equations for optimal intertemporal consumption of perishable and durable goods, respectively:

$$c_t^{-\gamma} = \beta R \mathbb{E}_t \left[c_{t+1}^{-\gamma}\right] + \mu_t, \tag{9}$$

$$c_t^{-\gamma} q_t = \nu h_t^{-\gamma_h} + \beta \mathbb{E}_t \left[c_{t+1}^{-\gamma} q_{t+1}\right] + (s + s_t) \frac{\mathbb{E}_t \left[q_{t+1}\right]}{R} \mu_t. \tag{10}$$

[4] We also considered a formulation of the LTV ratio as $sg(s_t)$, where $g(s_t) \equiv \exp\left(-\frac{1}{2}\sigma_s^2\right)\exp(s_t)$, where the first term in g cancels the average level effect on the LTV ratio introduced by a log-normal specification of s_t. This has no qualitative implications for our baseline result. However, changing s in this alternative formulation (for sensitivity analysis purposes) would also change the variability of the LTV ratio.

[5] A body of research on 'sudden stops' follows Mendoza (2010), where a credit constraint does not bind in the steady state. He achieves a determinate steady state by adopting Epstein (1983) preferences, where discounting is a function of past consumption, and calibrates this function such that the credit constraint does not bind in the steady state. See Schmitt-Grohé and Uribe (2003) on indeterminacy problems and resolutions in small open economy models with incomplete markets.

3 Equilibrium and solution procedure

The market for durables is simplified by assuming that supply is constant, i.e.

$$h_t = h > 0, \qquad t = 0, 1, 2, \ \dots \ , \infty \,, \tag{11}$$

holds in all periods. Applying (11), we can then state:

Definition 1 *An equilibrium is a set of functions d, c, q and μ that, conditional on d_{t-1} and $z_t \equiv [e_t, s_t]$, satisfy (2), (4), (9), (10). An equilibrium therefore satisfies*

$$c\left(d_{t-1}, z_t\right) + Rd_{t-1} = yf\left(e_t\right) + d\left(d_{t-1}, z_t\right), \tag{12}$$

$$c\left(d_{t-1}, z_t\right)^{-\gamma} = \beta R \mathbb{E}_t\left[c\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)^{-\gamma}\right] + \mu\left(d_{t-1}, z_t\right), \tag{13}$$

$$\begin{aligned} c\left(d_{t-1}, z_t\right)^{-\gamma} q\left(d_{t-1}, z_t\right) \ &= \ \nu h^{-\gamma_h} + \beta \mathbb{E}_t\left[c\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)^{-\gamma} q\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)\right] \\ &\quad + \left(s + s_t\right) \frac{\mathbb{E}_t\left[q\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)\right]}{R} \mu\left(d_{t-1}, z_t\right), \end{aligned} \tag{14}$$

$$\mu\left(d_{t-1}, z_t\right)\left[d\left(d_{t-1}, z_t\right) - \left(s + s_t\right) \frac{\mathbb{E}_t\left[q\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)\right] h}{R}\right] = 0, \tag{15}$$

where (15) is the complementary slackness condition associated with (4) and $\mu\left(d_{t-1}, z_t\right) \geq 0$, and where the exogenous disturbances, z_t, evolve according to (3) and (5).

Note that the exogenous stochastic variables e_t and s_t enter the equilibrium conditions (12)–(15) so that, when considering different mean-preserving spreads of each shock (Rothschild and Stiglitz, 1970, 1971), we do not introduce arbitrary exogenous mean level effects. Exogenous level effects and their accompanying biases have long been acknowledged in the literature of uncertainty shocks in business cycles; see, e.g., Rankin (1994).[6]

We solve the non-linear system (12)–(15) numerically. The state space spanned by d_{t-1} and z_t is discretized by 2,501 points for debt, and a five-state Markov chain for each of the shocks. Through Euler-equation iteration we obtain approximate policy functions $d_t = d\left(d_{t-1}, z_t\right)$, $c_t = c\left(d_{t-1}, z_t\right)$, $q_t = q\left(d_{t-1}, z_t\right)$ and $\mu_t = \mu\left(d_{t-1}, z_t\right)$. The recursive nature of the policy functions enables us to solve for the value function $V_t \equiv V\left(d_{t-1}, z_t\right) = \left[1/\left(1-\gamma\right)\right] c\left(d_{t-1}, z_t\right)^{1-\gamma} +$

[6] See also Lester *et al.* (2014), in some of their extensions of Cho *et al.* (2015). There, some exogenous level effects are present and acknowledged.

$\left[1/\left(1-\gamma^h\right)\right] h^{1-\gamma_h} + \beta \mathbb{E}_t \left[V\left(d\left(d_{t-1}, z_t\right), z_{t+1}\right)\right]$, which will be the basis for welfare analyses. The solution algorithm is detailed in Appendix B.

4 Cyclical welfare gains

We first turn to our choice of the model parameters. We deliberately abstain from calibrating the model to match the business cycle moments of any particular small open economy, for two main reasons. First, it is not our purpose to examine any particular country and, second, the model is too simple to mimic the business-cycle properties of a given economy. Instead, as part of the study of the key mechanism at work in the model, sensitivity analyses will be performed, which show that our result holds for a broad range of plausible parameterizations, but also reveal when it does not.

One period is interpreted as a quarter, such that $R = 1.01$ provides the commonly assumed 4% yearly real interest rate. To provide a role for borrowing, we assume that households are more 'impatient' than the financial markets, and set $\beta = 0.97$; cf. Reyes-Heroles and Tenorio (2017), among others. The average LTV ratio is $s = 0.7$, which is broadly in line with Calza *et al.* (2013) and Liu *et al.* (2013). Households' coefficients of relative risk aversion are set at a fairly standard $\gamma = \gamma_h = 2$; see, e.g., De Santis (2007), Onatski and Williams (2010), Benigno *et al.* (2013), and Sosa-Padilla (2018). Both the steady-state income and the stock of durables are normalized to 1. The price of durables is then determined by the preference parameter ν, which is set to 0.065, implying an average (stock of) durables-to-output ratio of about 0.95 in annual terms, and an annualized ratio of (household) debt-to-output ratio of 0.67, in line with values observed across various advanced economies, as documented by IMF (2017).

Regarding the shocks, we also choose rather conventional numbers. As for the income process, we assume a standard deviation of $\sigma_e = 0.02$, and an AR(1) parameter of $\rho_e = 0.95$. We set the same standard deviation for the LTV shock ($\sigma_s = 0.02$), while imposing slightly higher persistence ($\rho_s = 0.97$).[7]

[7] In quantitative analyses employing fully-fledged dynamic stochastic general equilibrium models, the volatility of the LTV shocks is found to be around three times higher relative to income volatility (see, e.g., Liu *et al.*, 2013). In our setting, however, this would result in situations characterized by negative consumption. To avoid this, we therefore opt for a rather conservative value of σ_s. As seen below, this works against our main findings, as the welfare gain of business cycles increases with σ_s in our robustness analysis.

Table 1. Baseline parameter values

Parameter	Description	Value
R	Gross real rate of interest	1.01
β	Discount factor	0.97
γ	CRRA, perishable consumption utility	2
γ_h	CRRA, durable consumption utility	2
ν	Utility weight, durable consumption	0.065
s	Average LTV ratio	0.7
y	Average income	1
h	Supply of durables	1
σ_e	Unconditional variance of the income shock	0.02
ρ_e	Autoregressive parameter of the income shock	0.95
σ_s	Unconditional variance of the financial shock	0.02
ρ_s	Autoregressive parameter of the financial shock	0.97

4.1 The welfare effects of business fluctuations

To measure the welfare costs of business cycles, we follow Lucas (1987) and ask by what percentage the stochastic consumption path should be increased to obtain the same unconditional welfare as in an economy with no shocks. As shown in Appendix C, this number is given by

$$\lambda = 100 \left[\left(\frac{\mathbb{E}\left[\overline{V}\left(d_{t-1}\right)\right] - u^h}{\mathbb{E}\left[V\left(d_{t-1}, z_t\right)\right] - u^h} \right)^{\frac{1}{1-\gamma}} - 1 \right], \tag{16}$$

where $\overline{V}\left(d_{t-1}\right)$ denotes equilibrium welfare in an economy with no shocks, and where $u^h \equiv \left[\nu / \left(1-\gamma_h\right)\right] h^{1-\gamma_h}$. With this welfare metric in mind, the unconditional cost of business cycles amounts to -0.24% of consumption, i.e., a net welfare gain. While not being large in absolute value, this is much larger than Lucas's (1987) original number(s), though it is the existence of such a gain that is of most interest to our analysis.

It is insightful to condition the welfare measure on the stock of debt and the shock realizations. To this end, the following measure of *conditional* welfare loss of business cycles can be derived (see Appendix C for further details):

$$\lambda^c\left(d_{t-1}, z_t\right) = 100 \left[\left(\frac{\overline{V}\left(d_{t-1}\right) - u^h}{\mathbb{E}_t V\left(d_{t-1}, z_t\right) - u^h} \right)^{\frac{1}{1-\gamma}} - 1 \right]. \tag{17}$$

The first panel of Figure 1 shows that, irrespective of the history of debt, when shocks take on their average values, then $\lambda^c < 0$. Hence, the presence of business cycles is welfare enhancing, particularly when initial debt is close to the deterministic steady state—even in this case, the gain is about a quarter of a percent of steady-state consumption—and, therefore, the economy is prone to switching to a regime in which the constraint does not bind.

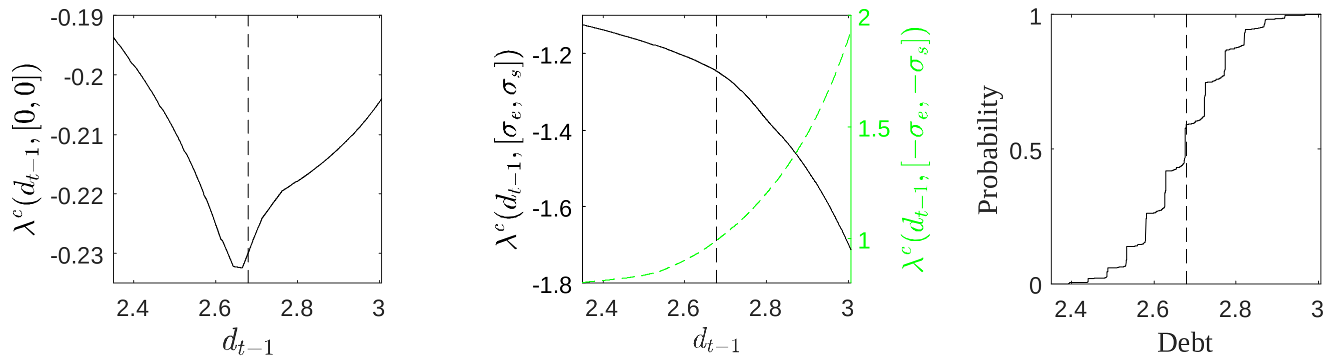

Figure 1: Conditional welfare losses and the stationary debt distribution. Left panel: Both shocks are initially at their means. Center panel: Both shocks are initially one s.d. higher (left axis, solid line), or one s.d. lower (right axis, green-dashed line) than their means. Right panel: Stationary cumulative distribution of debt. The vertical-dashed line denotes the deterministic steady-state debt. All parameters are at their baseline values.

Now, consider the central panel of Figure 1. Here we examine two opposite initial conditions. A 'bad' state, where both exogenous shocks are one standard deviation below their means, and a 'good' state, where both shocks are one standard deviation above. Notice that, irrespective of initial debt, the cost of business cycles is positive, conditional on a bad economic state. In fact, the magnitude of the cost can be conspicuous for an economy in high debt (0.5–2% of steady-state consumption). In the good state, instead, the opposite holds true: Irrespective of initial debt, we appreciate a business cycle gain, which increases over the support of d_{t-1} (1.1–1.7% of steady-state consumption).

In light of this asymmetry—and to ascertain the origins of the welfare gain of business fluctuations—it is important to quantify the chances that financial leverage is endogenously driven to a 'costly' region of its support. In this respect, the third panel of Figure 1 reports the stationary cumulative distribution function of debt. The vertical line corresponds to mean debt in the deterministic economy, which amounts to 2.6784.[8] Two insights are offered: First, the distribution of debt is rather narrow around the deterministic level; second, the distribution is skewed to the left, so that 58% of the time d_t is lower than its counterpart in the deterministic case. Hence, it is relatively rare that debt may actually end up in the region where business cycles are very costly, as implied by both our computation of λ and the first panel of Figure 1. The next subsection will be devoted to understanding the key driver of

[8] In the stochastic economy, instead, the mean is 2.6756: This slightly lower figure arises from precautionary saving, and in itself would not necessarily give rise to cyclical gains. This difference also explains why λ^c reaches its trough slightly to the left of the deterministic steady-state debt level in the left panel of Figure 1.

these results.

4.2 Why are business cycles beneficial?

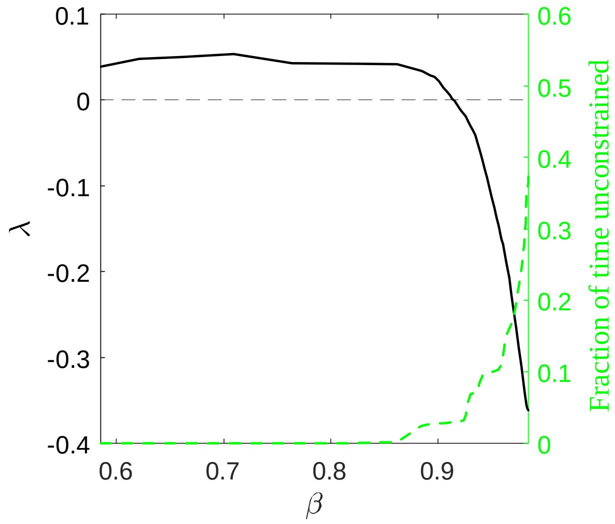

Figure 2: Welfare costs of business cycles for different discount factors. All other parameters are at their baseline values.

We now turn to dissecting the key mechanism behind the welfare gain from business fluctuations. To this end, we perform a series of exercises based on varying some key parameters, one at the time. It is particularly instructive to start by studying the impact of households' patience on unconditional welfare, as this has a tangible impact on consumers' attitude to smooth consumption over time. To this end, we examine welfare over a wide range of βs.

Figure 2 shows that, as consumers start with an implausibly high degree of impatience, the economy with uncertainty is welfare-dominated by the certainty scenario. Effectively, the endogenous switching effect is shut off when consumers are very impatient, so that only the traditional fluctuations effect is at work. The credit constraint binds strongly, such that shocks never lead to the occurrence of episodes where agents are unconstrained; cf. the dashed-green line.

However, as β increases beyond a certain threshold—which lies well below the range of values typically considered in calibrations based on quarterly data—the cost of business cycles eventually translates into a steadily increasing gain. This can be explained upon the fact that, as agents' desire to smooth consumption increases with their degree of patience, we observe an increasing frequency of episodes in which the credit constraint does not bind.

The tension between financial tightness and the intensity of a precautionary saving motive—

with the latter going beyond the one induced by prudence alone—is central to our story. To appreciate it, consider households' consumption Euler equation (9), forward it one period and eliminate $c_{t+1}^{-\gamma}$, so as to obtain:

$$c_t^{-\gamma} = \mu_t + (\beta R)^2 \mathbb{E}_t \left[c_{t+2}^{-\gamma}\right] + \beta R \mathbb{E}_t \left[\mu_{t+1}\right]. \tag{18}$$

Note that μ_t—which indexes the degree of tightness of the financial constraint—has a negative impact on current consumption, as the consumer borrows less, so as to attain lower consumption, in the presence of a credit constraint. As discussed above, the optimal intertemporal consumption behavior is characterized by precautionary saving, which arises from two sources. The first one is prudence, and emanates from $(\beta R)^2 \mathbb{E}_t \left[c_{t+2}^{-\gamma}\right]$. As uncertainty about future consumption increases (due to any type of underlying shock), the expected marginal utility of consumption also increases, as $\gamma > 0$. In addition to this standard fluctuations effect, our setting features another source of precautionary saving. Shocks cause variations in the tightness of the financial constraint: as $\mu_{t+1} \geq 0$, it must be that $\mathbb{E}_t \left[\mu_{t+1}\right] > 0$ when μ_{t+1} varies. As implied by (18), this increases the marginal cost of current borrowing. In turn, households save to reduce the risk of being credit constrained in the future. This source of precautionary saving drives the emergence of the endogenous switching effect, as households increase the chance of facing a non-binding credit constraint in the future; i.e., the chance of switching into a different, and more favorable, economic regime.[9]

With these two effects in mind, the role of increasing risk aversion can be appreciated from the left panel of Figure 3. Notably, when consumers are risk-neutral ($\gamma = 0$), the borrowing constraint is always binding and λ is virtually zero. When households become slightly risk-averse, fluctuations become costly, albeit very little (peaking at $\lambda \approx 0.004$ when $\gamma = 0.17$).[10] As γ increases, both sources of precautionary saving become stronger. However, compared with a model where only prudence drives savings, in the present setting higher risk aversion eventually drives consumption to a regime compatible with the financial constraint not being binding; and it does so more and more frequently. This entails a benefit to the consumer, who smooths consumption beyond what she is able to do when constrained; thus achieving

[9] Note that the endogenous switching effect is present irrespective of the chosen form of the utility function. The fluctuations effect would disappear with quadratic utility. In this case, (18) becomes $c_t = -\mu_t + (\beta R)^2 \mathbb{E}_t \left[c_{t+2}\right] - \beta R \mathbb{E}_t \left[\mu_{t+1}\right]$, which only brings about precautionary saving through endogenous switching stemming from the $\beta R \mathbb{E}_t \left[\mu_{t+1}\right]$ term.

[10] As illustrated in Figure D.1 in Appendix D, the welfare cost of fluctuations arising at low values of γ is driven by the presence of income shocks, as it arises also when we shut off financial shocks ($\sigma_s = 0$). In the absence of income shocks ($\sigma_e = 0$), however, business cycles are beneficial even as γ tends to zero.

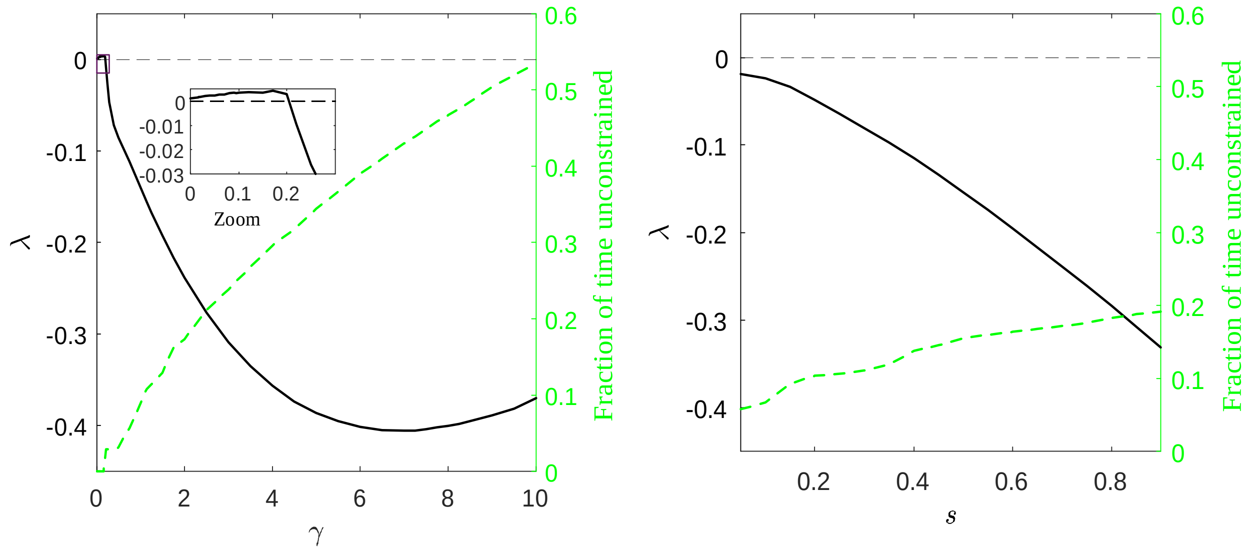

Figure 3: Welfare costs of business cycles for different values of relative risk aversion (left panel) and for different average LTV ratios (right panel). All other parameters are at their baseline values.

a gain from business cycles. As γ increases further, however, λ reverts its pattern, as the fluctuations effect gradually overcomes the endogenous switching effect.[11] In essence, the gain from further reducing the risk of being constrained eventually gets counteracted by the dislike of fluctuations *per se*, although the welfare measure remains negative for the range of conventional values of γ we consider.

Similar evidence emerges when examining the effects of increasing the LTV ratio. As depicted in the right panel of Figure 3, even at extremely low LTV ratios business fluctuations are beneficial. As s rises, a gradual relaxation of the constraint occurs, resulting in increasingly frequent episodes of slackness. In this case, however, the fluctuations effect never overcomes the endogenous switching effect, and λ displays a monotonically declining path.[12]

4.3 On the role of different shocks

The shocks at play in the model are important. To analyze their relative role, Figure 4 reports λ conditional on switching off either of them at the time. The left panel of the figure considers the case of no financial shocks ($\sigma_s = 0$): For an initial narrow range of values, the shocks hitting the economy are too small to make the financial constraint non-binding. In

[11] Notably, this happens well before the frequency of non-binding episodes approaches 1, at which point the endogenous switching effect is exhausted.

[12] In the absence of financial shocks, however, business cycles lead to a welfare loss at extremely low values of s, as shown in Figure D.2 in Appendix D. In that case, episodes of non-binding constraints are very infrequent.

the absence of the endogenous switching effect, we observe that $\lambda > 0$ (albeit marginally). Further increasing σ_e reinforces the endogenous switching effect over the fluctuations effect. However, as income fluctuations become very large, the precautionary motive becomes conspicuous, eventually compressing steady-state consumption too much, and driving λ into the 'costly' region.[13] By contrast, a rise in σ_s—absent income shocks ($\sigma_e = 0$)—translates into an increasing gain from business fluctuations (see the right panel of Figure 4).

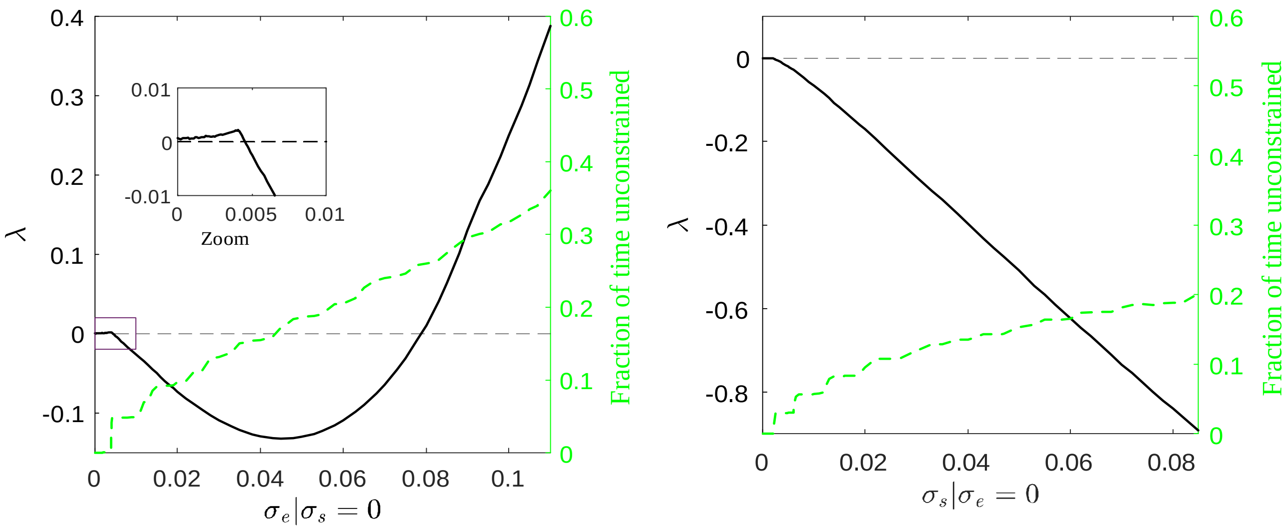

Figure 4: Welfare costs of business cycles for different standard deviations of a given shock, conditional the other shock being switched off. All the other parameters are at their baseline values.

The different role played by the two types of shocks is reminiscent of the result of Cho *et al.* (2015), who show that introducing multiplicative shocks implies that fluctuating economies may enjoy higher welfare, as compared with their no-shock counterparts. A mean effect of uncertainty is at work in their case—along with the usual fluctuations effect—as compared with situations in which shocks enter additively: Multiplicative shocks have the potential to raise the mean output and/or consumption, allowing consumers to take advantage of uncertainty by working harder and investing more during expansionary periods. By contrast, when uncertainty enters the economy additively, it has no beneficial effect on the choices that can be adjusted to it. This is the case for our income shock, which enters additively in the budget constraint. By contrast, the LTV shock affects the Euler equation for durables,

[13] The experiment with no financial shocks ($\sigma_s = 0$) sheds light on the role of the endogenous financial accelerator. Specifically, in the special case of a constant credit limit (i.e., without financial shocks and with a constant asset price, $q_t = q, \forall t$), we observe that fluctuations are costly under our baseline calibration. Under these circumstances, very large income shocks are required to make households unconstrained. This implies that the endogenous switching effect only comes into play when the fluctuations effect is already very strong. This is consistent with the findings of İmrohoroğlu (1989) in a model with an Aiyagari (1994)-style constant borrowing limit, which by construction does not embody the financial accelerator.

(10), so that consumers can take advantage of periods of relatively lax financial conditions to smooth consumption. It is important to stress that this mean effect shall not be confused with the endogenous switching effect, but rather is complementary to it. This can be seen from the fact that we still observe substantial welfare gains when the financial (and multiplicative) shock is switched off. In fact, as indicated by the left panel of Figure 4, abstracting from the mean effect implies that the endogenous switching effect channeled through the (additive) income shock alone still accounts for about 30% of the gain from business fluctuations at the baseline calibration (i.e., 0.07/0.24).[14]

5 Concluding remarks

This paper considers the welfare cost of business cycles in a credit economy with collateralized debt. Welfare tends to be higher in the economy with aggregate fluctuations, as compared with the benchmark model without uncertainty, as shocks facilitate endogenous switching from 'bad' to 'good' economic regimes. This benefit may outweigh the conventional losses due to risk aversion.

This insight has key implications for stabilization policies. Seeking to reduce business cycle volatility may not be beneficial, especially in situations where credit standards are relatively lax and/or volatile, so that collateral constraints may become slack sufficiently often. In both cases, there are gains from business fluctuations, as agents can at least temporarily behave as standard consumption smoothers. By contrast, in contexts where circumstances limit the likelihood of households ending up in a financially unconstrained regime, the standard fluctuations effect due to uncertainty makes policy intervention meaningful, yet not strictly necessary—due to a negligible welfare cost—as in the standard Lucas (1987) argument.

[14] Otherwise, while switching off the income shock does not tell us much about the relative contribution of the mean and the endogenous switching effect, it also shows that financial shocks exert most of the contribution in terms of generating the gain from business cycles.

References

Aiyagari, S. R., 1994, Uninsured idiosyncratic risk and aggregate saving, Quarterly Journal of Economics 109, 659–684.

Alvarez, F. and U. J. Jermann, 2004, Using asset prices to measure the cost of business cycles, Journal of Political Economy 112, 1223–1256.

Barlevy, G., 2004, The cost of business cycles under endogenous growth, American Economic Review, 94, 964–990.

Barro, R. J., 2006, On the welfare costs of consumption uncertainty, working paper, Harvard University.

Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. R. Young, 2013, Financial crises and macro-prudential policies, Journal of International Economics 89, 453–470.

Bianchi, J., 2011, Overborrowing and systemic externalities in the business cycle, American Economic Review 101, 3400–26.

Bianchi, J. and E. G. Mendoza, 2018, Optimal time-consistent macroprudential policy, Journal of Political Economy 126, 588–634.

Boz, E. and E. G. Mendoza, 2014, Financial innovation, the discovery of risk, and the U.S. credit crisis, Journal of Monetary Economics 62, 1–22.

Calza, A., T. Monacelli and L. Stracca, 2013, Housing finance and monetary policy, Journal of the European Economic Association 11, 101–22.

Cho, J.-O., T. F. Cooley and H. S. Kim, 2015, Business cycle uncertainty and economic welfare, Review of Economic Dynamics 18, 185–200.

De Santis, M., 2007, Individual consumption risk and the welfare cost of business cycles, American Economic Review 97, 1488–1506.

Eggertsson, G. B., and P. Krugman, 2012, Debt, deleveraging, and the liquidity trap: A Fisher-Minsky-Koo approach, The Quarterly Journal of Economics 127, 1469–1513.

Epstein, L., 1983, Stationary cardinal utility and optimal growth under uncertainty, Journal of Economic Theory 31, 133–152.

Galí, J., M. Gertler, and J. D. Lopez-Salido, 2007, Markups, gaps, and the welfare costs of economic fluctuations, Review of Economics and Statistics 89, 44–59.

Iacoviello, M., 2005, House prices, borrowing constraints and monetary policy in the business cycle, American Economic Review 95, 739–764.

IMF, 2017, Global financial stability report (October): Is growth at risk?, International Monetary Fund, Washington D.C.

İmrohoroğlu, A., 1989, Cost of business cycles with indivisibilities and liquidity constraints, Journal of Political Economy 97, 1364–1383.

Jermann, U. and V. Quadrini, 2012, Macroeconomic effects of financial shocks, American Economic Review 102, 238–71.

Jeanne, O. and A. Korinek, 2010, Managing credit booms and busts: A Pigouvian taxation approach, NBER Working Paper 16377.

Jones, C., V. Midrigan and T. Philippon, 2018, Household leverage and the recession, mimeo, International Monetary Fund and New York University.

Judd, K. L., 1988, Numerical methods in economics, The MIT Press.

Justiniano, A., G. Primiceri, A. Tambalotti, 2015, Household leveraging and deleveraging, Review of Economic Dynamics 18, 3–20.

Kiyotaki, N. and J. Moore, 1997, Credit cycles, Journal of Political Economy 105, 211–248.

Kimball, M. S., 1990, Precautionary saving in the small and in the large, Econometrica 58, 53–73.

Kopecky, K. A. and R. M. H. Suen, 2010, Finite state Markov-chain approximations to highly persistent processes, Review of Economic Dynamics 13, 701–714.

Krusell, P. and A. Smith Jr., 1999, On the welfare effects of eliminating business cycles, Review of Economic Dynamics 2, 245–272.

Krusell, P., T. Mukoyama, A. Şahin and A. A. Smith Jr., 2009, Revisiting the welfare effects of eliminating business cycles, Review of Economic Dynamics 12, 393–404.

Lester R., M. Pries and E. Sims, 2014, Volatility and welfare, Journal of Economic Dynamics & Control 38, 17–36.

Liu, Z., P. Wang and T. Zha, 2013, Land-price dynamics and macroeconomic fluctuations, Econometrica 81, 1147–84.

Liu, Z. and P. Wang, 2014, Credit constraints and self-fulfilling business cycles, American Economic Journal: Macroeconomics 6, 32–69.

Lucas Jr., R. E., 1987, Models of business cycles, Basil Blackwell, New York.

Mendoza, E. G., 2010, Sudden stops, financial crises, and leverage, American Economic Review 100, 1941–1966.

Obstfeld, M., 1994, Evaluating risky consumption paths: the role of intertemporal substitutability, European Economic Review 38, 1471–1486.

Onatski, A. and N. Williams, 2010, Empirical and policy performance of a forward-looking monetary model, Journal of Applied Econometrics 25, 145–176.

Rankin, N., 1994, Monetary uncertainty in discrete-time utility-of-money models, Economics Letters 44, 127–132.

Reis, R., 2007, The time-series properties of aggregate consumption: Implications for the costs of fluctuations, Journal of the European Economic Association 7, 722–753.

Reyes-Heroles, R. and G. Tenorio, 2017, Managing capital flows in the presence of external risks, International Finance Discussion Papers 1213.

Rendahl, P., 2015, Inequality constraints and Euler equation-based solution methods, The Economic Journal 125, 1110–35.

Rothschild, M. and J. E. Stiglitz, 1970, Increasing risk I: A definition, Journal of Economic Theory 2, 225–43.

Rothschild, M. and J. E. Stiglitz, 1971, Increasing risk II: It's economic consequences, Journal of Economic Theory 3, 66–84.

Rouwenhorst, K. G., 1995, Asset pricing implications of equilibrium business cycle models. In: Cooley, T. F. (ed.), Frontiers of business cycle research. Princeton University Press, Princeton, 294–330.

Schmitt-Grohé, S. and M. Uribe, 2003, Closing small open economy models, Journal of International Economics 61, 163–185.

Schmitt-Grohé, S. and M. Uribe, 2016, Multiple equilibria in open economy models with collateral constraints: overborrowing revisited, NBER Working Paper, 22264, Cambridge, Massachusetts.

Storesletten, K., C. I. Telmer and A. Yaron, 2001, The welfare costs of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk, European Economic Review 45, 1311–39.

Sosa-Padilla, C., 2018, Sovereign defaults and banking crises, Journal of Monetary Economics 99, 88–105.

Appendices

A Deterministic steady state

In the absence of any shocks, we have from (13) that

$$\begin{aligned} \mu &= (c)^{-\gamma} - \beta R \mathbb{E}\left[(c)^{-\gamma}\right], \\ &= (c)^{-\gamma}(1-\beta R) > 0, \end{aligned} \tag{A.1}$$

where undated variables denote deterministic steady-state values, and where the inequality sign follows from $\beta < 1/R$. From (15) we therefore get

$$d = s\frac{qh}{R}. \tag{A.2}$$

From (12) and (A.2) we get

$$c + sqh\left(1 - R^{-1}\right) = y. \tag{A.3}$$

By (14), we get

$$\nu(h)^{-\gamma_h} = (c)^{-\gamma} q - \beta q c^{-\gamma} - s\frac{q}{R}\mu,$$

which by (A.1) becomes

$$\begin{aligned} \nu(h)^{-\gamma_h} &= (c)^{-\gamma} q - \beta q (c)^{-\gamma} - s\frac{q}{R}(c)^{-\gamma}(1-\beta R), \\ &= (c)^{-\gamma} q\left[1 - \beta - s\frac{1}{R}(1-\beta R)\right]. \end{aligned} \tag{A.4}$$

Equations (A.3) and (A.4) provide the unique solutions for c and q. Conditional on these, closed-form solutions for μ and d follow from (A.1) and (A.2), respectively.

B Solution algorithm

We solve the model numerically through Euler-equation iteration on the policy functions. Our problem is non-standard because the credit constraint introduces a state-dependent discontinuity. As argued by Rendahl (2015), searches for solutions can in such cases be divergent, cyclical, or even non-convergent. We therefore follow Judd (1988) and introduce 'dampening' parameters in the updating of policy functions. This implies that in any update of a policy function, only a fraction of the new function will replace the old. This fosters convergence.

We first discretize the state variables d_{t-1}, e_t, and s_t such that $d_{t-1} \in \mathbf{d}_{t-1} \equiv [d_{\min}, \ldots, d_{\max}]^{\mathsf{T}}$, $e_t \in \mathbf{e}_t \equiv [e_{\min}, \ldots, e_{\max}]^{\mathsf{T}}$, $s_t \in \mathbf{s}_t \equiv [s_{\min}, \ldots, s_{\max}]^{\mathsf{T}}$. In the construction of $\mathbf{d}_{t-1}$ we allow for a relatively finer grid around the deterministic steady state to foster precision. In the construction of state vectors we secure that the model does not imply starvation for high initial debt combined with sufficiently adverse shocks. The discretization of the shocks uses Rouwenhorst's (1995) method of approximating AR(1) processes by Markov chains with transition matrices, $\mathbf{P}_{\mathbf{e}}$ and $\mathbf{P}_{\mathbf{s}}$, respectively. We thereby follow Kopecky and Suen (2010), who find that this method best approximates very persistent processes compared with other methods. To simplify notation and computations, we create a column vector of all shock combinations, $\mathbf{z}_t \equiv \text{vec}\left(\mathbf{s}_t\mathbf{e}_t^{\mathsf{T}}\right)$. The associated transition matrix for $\mathbf{z}_t$ is then given by $\mathbf{\Pi} \equiv \mathbf{P}_{\mathbf{e}} \otimes \mathbf{P}_{\mathbf{s}}$, where $\otimes$ is the Kronecker product. We use 2,501 debt states and five states for each shock.

In the solution procedure we form a matrix of all state combinations, $\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}$, and seek solutions for policy functions yielding matrices $c\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, $q\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, $d\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, and $\mu\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, which satisfy the equilibrium conditions. Note that in any state, we have either $\mu_t = 0$ or $\mu_t > 0$. Call these two cases the *unconstrained* and *constrained* regime, respectively. In each iteration we solve the model in two blocks—one for each regime. This exploits the different structure of the solution in either regime.[15] Subsequently, the policy matrices are appropriately merged before proceeding with the next iteration. The algorithm is as follows:

1. Make initial guesses $c^0\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$ and $q^0\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$.

2. Use (12) to obtain $d\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = c^i\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) + R\widetilde{\mathbf{d}}_{t-1} - yf\left(\widetilde{\mathbf{e}}_t\right)$, where $\widetilde{\mathbf{d}}_{t-1}$ is a matrix of repeated columns of $\mathbf{d}_{t-1}$ conformable with $\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}$. The income shocks are used to construct the matrix $f\left(\widetilde{\mathbf{e}}_t\right)$, which has identical row vectors of the possible income-shock values, conformable with $\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}$.

3. Use $\mathbf{d}_t = d\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$ to compute $\mathbf{c}_{t+1} = \widehat{c}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)$ and $\mathbf{q}_{t+1} = \widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)$ through column-wise interpolation on $\mathbf{d}_{t-1}$ and $c^i\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$ and $q^i\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)$, respectively.

4. Derive policy functions in the *unconstrained* regime:

   (a) By definition, $\mu^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = 0$.

   (b) From (13),
   $$c^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = \left\{\beta R\left[\widehat{c}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)^{-\gamma}\mathbf{\Pi}^{\mathsf{T}}\right]\right\}^{-1/\gamma}.$$

   (c) From (14),
   $$\begin{aligned} q^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) &= c^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)^{\gamma} \\ &\circ\left\{\nu\left(h\right)^{-\gamma_h} + \beta\left[\left(\widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)\circ\widehat{c}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)^{-\gamma}\right)\mathbf{\Pi}^{\mathsf{T}}\right]\right\}, \end{aligned}$$
   where $\circ$ denotes element-by-element multiplication.

   (d) By (12), find $d^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) = c^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) + R\widetilde{\mathbf{d}}_{t-1} - yf\left(\widetilde{\mathbf{e}}_t\right)$.

5. Derive policy functions in the *constrained* regime:

   (a) Let the matrix $\widetilde{\mathbf{s}}_t$ contain identical row vectors of the possible LTV-shock values, conformable with $\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}$. In each column of $\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}$ identify the states where
   $$d^{uncon}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right) > \left[\left(s+\widetilde{\mathbf{s}}_t\right)/R\right]\circ\left[\widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)\mathbf{\Pi}^{\mathsf{T}}\right]h,$$
   as these violate (4) and therefore characterize the constrained regime. For any matrix $\mathbf{X}_t$, denote by $\left[\mathbf{X}_t\right]^j$ the j^{th} column of $\mathbf{X}_t$ only consisting of such identified states.

   (b) From (4), when the constraint binds,
   $$\left[d^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^j = \left(\left[s+\widetilde{\mathbf{s}}_t\right]^j/R\right)\circ\left[\widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^{\mathsf{T}}\right)\mathbf{\Pi}^{\mathsf{T}}\right]^j h, \quad \text{all } j.$$

   (c) From (15),
   $$\left[c^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^j = y\left[f\left(\widetilde{\mathbf{e}}_t\right)\right]^j - R\left[\widetilde{\mathbf{d}}_{t-1}\right]^j + \left[d^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^{\mathsf{T}}\right)\right]^j, \quad \text{all } j.$$

[15] This approach is in accordance with the one suggested by Jeanne and Korinek (2010).

(d) From (13),

$$\left[\mu^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right]^j = \left(\left[c^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right]^j\right)^{-\gamma} - \beta R\left[\widehat{c}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^\mathsf{T}\right)^{-\gamma}\mathbf{\Pi}^\mathsf{T}\right]^j, \quad \text{all } j.$$

(e) From (14),

$$\begin{aligned}\left[q^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right]^j &= \left(\left[c^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right]^j\right)^{\gamma} \\ &\circ\left\{\nu\left(h\right)^{-\gamma_h} + \beta\left[\left(\widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^\mathsf{T}\right)\circ\widehat{c}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^\mathsf{T}\right)^{-\gamma}\right)\mathbf{\Pi}^\mathsf{T}\right]^j\right. \\ &\left.+\left(\left[s+\widetilde{\mathbf{s}}_t\right]^j/R\right)\circ\left[\widehat{q}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^\mathsf{T}\right)^{-\gamma}\mathbf{\Pi}^\mathsf{T}\right]^j\circ\left[\mu^{con}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right]^j\right\}, \quad \text{all } j.\end{aligned}$$

6. An updated set of policy functions $c^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)$, $q^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)$, and the associated $d\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)$ and $\mu\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)$, are built from the respective matrices found in the unconstrained and constrained regimes. Specifically, in the policy matrices derived for the unconstrained regime, replace the values with the ones found in the constrained regime for the states identified in Step 5a.

7. If
$$\left\|\text{vec}\left[c^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right] - \text{vec}\left[c^{i}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right]\right\|_\infty < \varepsilon,$$
and
$$\left\|\text{vec}\left[q^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right] - \text{vec}\left[q^{i}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right]\right\|_\infty < \varepsilon,$$
where ε is some tolerance criterion, then stop (we use $\varepsilon = 10^{-8}$). Otherwise, update according to $c^{i+2}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right) = \omega_c c^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right) + (1-\omega_c)\, c^i\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)$ and $q^{i+2}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right) = \omega_q q^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right) + (1-\omega_q)\, q^i\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)$, where $0 < \omega_c,\ \omega_q < 1$ are dampening parameters, and go to 2.

Subsequently, the value function is computed. Start with a guess $V^0\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)$. Then proceed as follows:

1. Use $d\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)$ to obtain $V_{t+1} = \widehat{V}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^\mathsf{T}\right)$ through column-wise interpolation on $\mathbf{d}_{t-1}$ and $V^i\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)$.

2. Compute
$$V^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right) = \frac{1}{1-\gamma}\left[c\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right]^{1-\gamma} + \frac{\nu}{1-\gamma_h}\left(h\right)^{1-\gamma_h} + \beta\widehat{V}\left(\mathbf{d}_{t-1}\mathbf{z}_{t+1}^\mathsf{T}\right)\mathbf{\Pi}^\mathsf{T}.$$

3. If
$$\left\|\text{vec}\left[V^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right] - \text{vec}\left[V^{i}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)\right]\right\|_\infty < \varepsilon,$$
where ε is the tolerance criterion, then stop. Otherwise, set $V^{i+2}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right) = \omega_V V^{i+1}\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right) + (1-\omega_V)\, V^i\left(\mathbf{d}_{t-1}\mathbf{z}_t^\mathsf{T}\right)$, where $0 < \omega_V < 1$ is a dampening parameter, and go to 1.

C Derivation of the costs of business cycles

We want to find the value of λ that secures $\mathbb{E}\left[V\left(d_{t-1}, z_t\right)\right] = \mathbb{E}\left[\overline{V}\left(d_{t-1}\right)\right]$; i.e., indifference between the stochastic and non-stochastic economies. Using the definitions of the value functions together with the definition of λ as the percentage increase in the consumption path in the stochastic economy to secure indifference, λ satisfies

$$\begin{aligned} & \mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\left(\frac{1}{1-\gamma}\left[(1+\lambda/100)\,c\left(d_{t-1},z_t\right)\right]^{1-\gamma}+\frac{\nu}{1-\gamma_h}h^{1-\gamma_h}\right)\right] \\ = \; & \mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\left(\frac{1}{1-\gamma}\left[\overline{c}\left(d_{t-1}\right)\right]^{1-\gamma}+\frac{\nu}{1-\gamma_h}h^{1-\gamma_h}\right)\right], \end{aligned} \tag{C.1}$$

where $\overline{c}\left(d_{t-1}\right)$ is the policy function for consumption under certainty. From (C.1) we readily obtain

$$(1+\lambda/100)^{1-\gamma}\,\mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\frac{1}{1-\gamma}\left[c\left(d_{t-1},z_t\right)\right]^{1-\gamma}\right]=\mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\frac{1}{1-\gamma}\left[\overline{c}\left(d_{t-1}\right)\right]^{1-\gamma}\right],$$

and therefore

$$\begin{aligned} (1+\lambda/100)^{1-\gamma} &= \frac{\mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\frac{1}{1-\gamma}\left[\overline{c}\left(d_{t-1}\right)\right]^{1-\gamma}\right]}{\mathbb{E}\left[\sum_{t=1}^{\infty}\beta^{t-1}\frac{1}{1-\gamma}\left[c\left(d_{t-1},z_t\right)\right]^{1-\gamma}\right]}, \\ &= \frac{\mathbb{E}\left[\overline{V}\left(d_{t-1}\right)\right]-u^h}{\mathbb{E}\left[V\left(d_{t-1},z_t\right)\right]-u^h}, \end{aligned} \tag{C.2}$$

where the second line in (C.2) follows from the definitions of the value functions and u^h. From (C.2), we immediately recover the unconditional welfare measure, as desired.

As for the conditional welfare measure, $\lambda^c\left(d_{t-1},z_t\right)$, this satisfies

$$\begin{aligned} & \mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\left(\frac{1}{1-\gamma}\left[\left(1+\lambda^c\left(d_{t-1},z_t\right)/100\right)c\left(d_{s-1},z_s\right)\right]^{1-\gamma}+\frac{\nu}{1-\gamma_h}h^{1-\gamma_h}\right)\right] \\ = \; & \mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\left(\frac{1}{1-\gamma}\left[\overline{c}\left(d_{s-1}\right)\right]^{1-\gamma}+\frac{\nu}{1-\gamma_h}h^{1-\gamma_h}\right)\right]. \end{aligned} \tag{C.3}$$

Similar manipulations as used above in the case of (C.1), readily yield (17).

D Additional figures

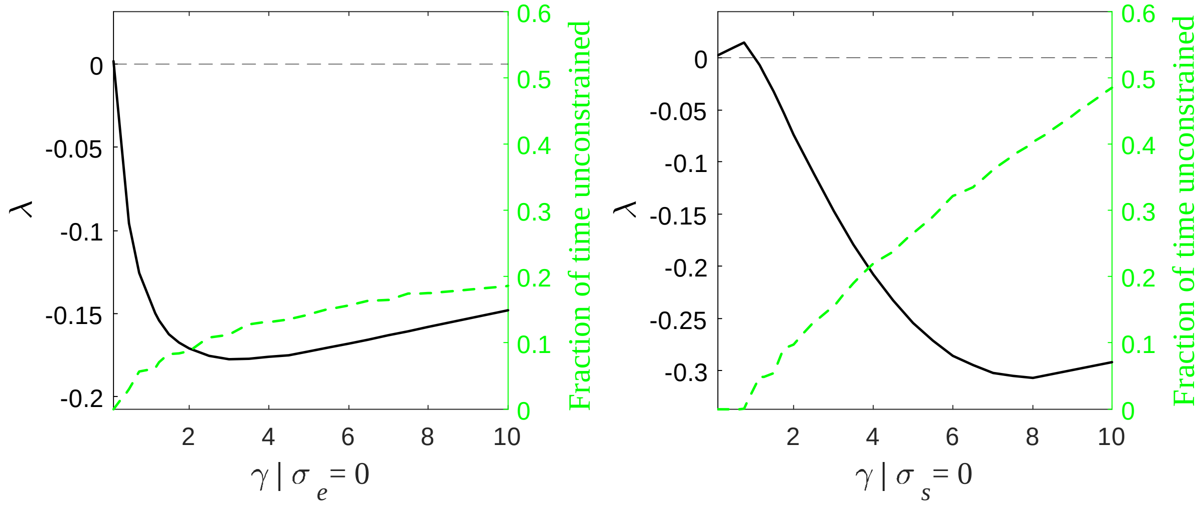

Figure D.1: Welfare costs of business cycles for different values of relative risk aversion, in the absence of income shocks (left panel) and LTV shocks (right panel). All the other parameters are at their baseline values.

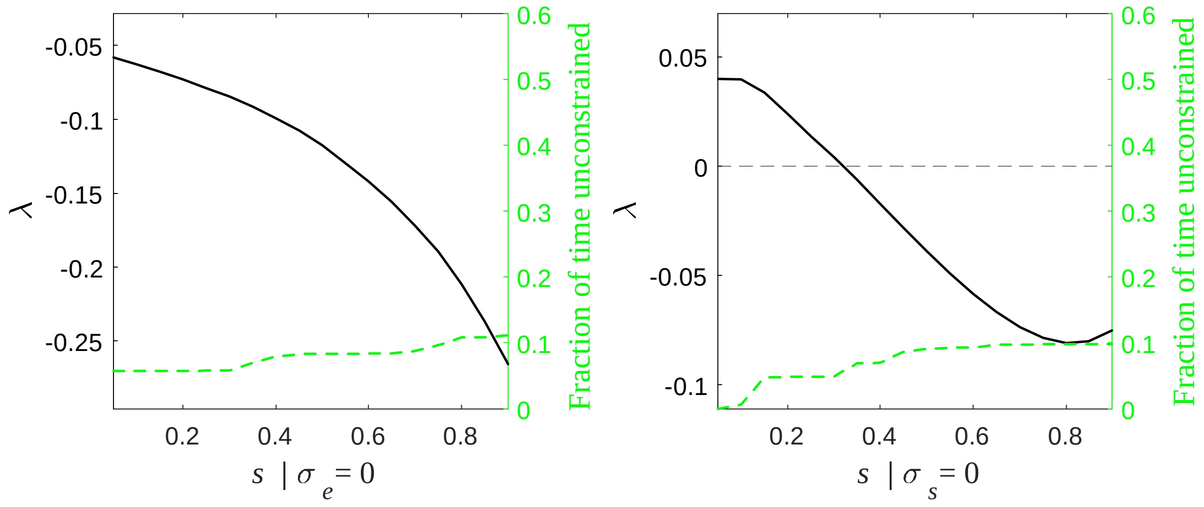

Figure D.2: Welfare costs of business cycles for different average LTV ratios in the absence of income shocks (left panel) and LTV shocks (right panel). All the other parameters are at their baseline values.