Abstract

In the last decades, capital markets across the industrialized world have undergone massive deregulation, involving increases in the loan-to-value (LTV) ratios of households and firms. We study the business-cycle implications of this phenomenon in a dynamic general equilibrium model with multiple credit-constrained agents. Starting from low LTV ratios, a progressive relaxation of credit constraints leads to both higher macroeconomic volatility and stronger comovement between debt and real variables. This pattern reverses at LTV ratios not far from those currently observed in many advanced economies, since credit constraints become non-binding more often. As expansionary shocks may make credit constraints non-binding, while contractionary shocks cannot, recessions become deeper than expansions. The non-monotonic relationship between credit market conditions and macroeconomic fluctuations poses a serious challenge for regulatory and macroprudential policies.

Keywords: Occasionally non-binding credit constraints, business cycles, capital-market liberalization, capital-market regulation.

JEL: E32, E44.
1 Introduction

Credit flows are crucial for the functioning of an economy where inhabitants want to alter the profile of purchases over time. Consumers may want to smooth consumption and finance their purchases of durable goods. Likewise, firms may desire to obtain funds for investment projects that only pay off later. Such intertemporal trades are typically plagued by informational problems leading to a multitude of financial market imperfections. One implication is that households and firms become credit constrained, and often have to provide collateral to obtain loans. Under such circumstances, the degree to which credit constraints bind is influential for the economy’s response to various disturbances. The main purpose of this paper is to study the business-cycle properties of a conventional DSGE model with credit constraints à la Kiyotaki and Moore (1997) under different credit conditions. Since the limits to credit acquisition are largely determined by how much an agent can borrow against his collateral—the loan-to-value (LTV) ratio—we model changes in credit conditions as changing LTV ratios.

Capital markets have undergone massive deregulation across the industrialized world in the past decades. Figure 1 shows loans relative to assets for households and firms in the US for the post-war period. The observed secular increases are consistent with increased credit availability through higher LTV ratios. The literature on credit limits and business cycles, however, is largely silent on the business cycle implications of substantial changes in the LTV ratios. We therefore contribute to filling this void, as much recent policy debate following the financial crisis has focused on limiting credit availability in general, or imposing countercyclical credit measures, e.g., IMF (2013).

We incorporate collateral constraints into a real business cycle model with heterogeneous agents in the vein of Iacoviello (2005), Liu et al. (2013), Justiniano et al. (2015); inter alia. A durable good, land, is used for both consumption purposes and production. In addition, land serves as collateral for “impatient”, credit-constrained households as well as for entrepreneurs. Entrepreneurs also invest in physical capital, which is used as collateral. The lenders in the economy are “patient”, non-constrained households. In contrast to most of the existing literature, we explore the implications of credit constraints not

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1As we discuss in Appendix A, the aggregate ratios of loans to assets reported in Figure 1 are likely to understate the actual LTV requirements faced by the marginal borrower. However, while alternative measures may yield higher levels of LTV ratios, they give rise to the same conclusions about the development over time of these ratios.
binding at all points in time.

Our main findings show that macroeconomic volatility and comovement between debt and real variables display a hump-shaped pattern in response to changes in the LTV ratio. Over most of the range of LTV ratios, higher credit limits allow financially constrained agents to succumb to their relative impatience and engage in debt-financed consumption and investment. This reinforces the macroeconomic repercussions of shocks that affect the borrowing capacity of these agents. As a result, output fluctuations become larger and credit issuance becomes more procyclical with a deepening of financial markets.\textsuperscript{2} Eventually, a further increase in the LTV ratio reverses this pattern—volatility and comovement, however, never return to the levels observed at low credit limits. Higher LTV ratios increase the likelihood that credit constraints become non-binding in the face of expansionary shocks. In such cases, the consumption and investment decisions of households and entrepreneurs are delinked from changes in the value of the collateral assets. As a result, positive shocks that bring about an expansion have a weaker impact on the economy than similar-sized negative shocks, leading to lower volatility and a negatively skewed business cycle.

\textsuperscript{2}These findings contrast with a strand of the literature that has attributed the observed drop in output volatility and in the comovement between real activity and debt since the mid-1980s to the wave of financial liberalization unraveling over the same period; see, e.g., Jermann and Quadrini (2009) and Campbell and Hercowitz (2009).
The non-monotonic relationship between credit-market conditions and macroeconomic volatility creates a trade-off for macroprudential policy. Lower credit limits may succeed in dampening the asset price sensitivity of intramarginal borrowers, who remain credit constrained before and after the intervention. Yet, on the margin, lower credit limits will increase the frequency by which credit constraints bind, subjecting borrowers more heavily to fluctuations in credit availability. For values of the LTV ratio in line with those currently observed in many advanced economies, the latter effect may well dominate, in which case a tightening of credit conditions will have the opposite effect on output volatility than intended.

A key feature of our model is that technology shocks are substantially amplified by the presence of collateral constraints. This is noteworthy, since previous studies have reported that collateral constraints have little or no role in amplifying and propagating shocks to firm productivity, see, e.g., Cordoba and Ripoll (2004) and Liu et al. (2013). The amplification relies on two main features. The first is the presence of land in the production technology, which ensures that a positive technology shock drives up the current and future marginal productivity of land.\textsuperscript{3} The second is the presence of two types of credit-constrained agents—impatient households and entrepreneurs—whose borrowing activity features a strategic complementarity: After a positive technology shock, the increase in the marginal productivity of land induces entrepreneurs to demand more of it. The resulting upward pressure on the land price leads to an increase in the value of both impatient households’ and entrepreneurs’ collateral assets, causing an endogenous relaxation of their credit constraints. Both agents therefore demand more land, causing further upward movements in expected future credit availability, and so on. This two-sided externality, which is stronger at relatively higher LTV ratios, ensures that more land ends up in the hands of the entrepreneurs, and thus in productive use.

At the same time, we show that shocks to land demand play only a minor role for output fluctuations, although they remain the main drivers of land prices. This contrasts with the findings of Liu et al. (2013), where entrepreneurial collateralized borrowing amplifies shocks to patient households’ land demand. In their setup these shocks emerge as a key driver of investment and, in turn, output fluctuations. In our model, instead,

\textsuperscript{3}Without land in the production function entrepreneurs would optimally hold zero land, so that their collateral value would be de-linked from the price of land.
shocks to land demand shift both patient and impatient households’ preferences, so that land tends to be ‘tied up’ away from productive use. This greatly reduces the impact of this shock on output. To that effect, the presence of a financially constrained household is particularly important, as a positive shock to impatient households’ preference for land services reinforces their attitude to accumulate land in order to access credit. Finally, it is important to stress that financial shocks in the form of exogenous variations in the LTV ratios faced by households and entrepreneurs account for a substantial share of output fluctuations under high LTV ratios in our setup. This is in line with the evidence of Jermann and Quadrini (2012), who report that financial shocks in the form of changes to the recovery ratio of assets in liquidation are important drivers of output fluctuations.

Our work is related to Guerrieri and Iacoviello (2014), who also exploit the asymmetric implications of occasionally binding credit constraints. They demonstrate that the macro-economic sensitivity to house price changes is much smaller during booms—when house prices are high and collateral constraints therefore likely to be non-binding—than during recessions, when credit constraints typically bind. In the model of Guerrieri and Iacoviello (2014) only households are credit constrained, as they focus on the role of the housing market in the US, whereas we consider a setup with credit constrained households and firms. In two recent contributions, Justiniano et al. (2014, 2015) have shown that higher credit limits have a modest impact on the macroeconomy, and thus are unlikely to explain the pre-crisis boom in the US economy. However, they only consider increases in the LTV ratio from an already high level (above 0.8), in which case our model also produces limited changes in output volatility.  

The rest of the paper is organized as follows. Section 2 presents our model, and the equilibrium and its computation are described in Section 3. Section 4 contains the presentation and discussion of our results. Finally, Section 5 concludes and offers some policy implications as well as directions for further research.

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4Walentin (2014) studies an increase in the LTV ratio from 0.85 to 0.95 in a model featuring construction of housing, and finds that this leads to an increase in output volatility of around 5%.
2 The model

We consider a real business cycle model with heterogeneous agents and credit limits in the vein of Iacoviello (2005), Liu et al. (2013), Justiniano et al. (2015); inter alia. The economy is populated by entrepreneurs and two types of households. Agents are differentiated by their discount factors. So-called patient households have the highest discount factor, effectively making them lenders in the economy. Impatient households and entrepreneurs, instead have lower discount factors, and can only borrow up to some proportion of the present value of their assets. Both patient and impatient households work, consume non-durable goods and (durable) land, where the latter can be interpreted as related to housing services. Entrepreneurs only consume non-durable goods, and accumulate both land and physical capital, which they rent to producers. These are modelled by a perfectly competitive sector, where firms of unit mass combine labor from both types of households as well as capital and land from entrepreneurs, so as to produce non-durable consumption goods and new capital goods. Agents are in total of unit mass, with impatient households and entrepreneurs being of mass $0 < n_I < 1$ and $0 < n_E < 1$, respectively.

2.1 Patient and impatient households

The preferences of the households are defined over non-durable consumption, $C_i^t$, the stock of land, $H_i^t$, and the fraction of time devoted to labor, $N_i^t$, where $i \in \{P, I\}$ refers to patient and impatient households, respectively. Household $i$ maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} (\beta^i)^t \left[ \frac{1}{1-\sigma_C^i} (C_i^t)^{1-\sigma_C^i} + \frac{\varepsilon_t}{1-\sigma_H^i} (H_i^t)^{1-\sigma_H^i} \right] + \frac{\nu^i}{1-\sigma_N^i} (1-N_i^t)^{1-\sigma_N^i} \right\}, \quad \nu^i > 0,$$

where $\varepsilon_t$ is a land-preference shock satisfying

$$\log \varepsilon_t = \log \varepsilon + \rho_{\varepsilon} (\log \varepsilon_{t-1} - \log \varepsilon) + u_t, \quad 0 < \rho_{\varepsilon} < 1,$$

where $\varepsilon > 0$ denotes the steady-state value of $\varepsilon_t$, and where $u_t \sim N(0, \sigma_{\varepsilon}^2)$. Moreover, $0 < \beta^i < 1$ is the discount factor, $\sigma_C^i > 0$, $\sigma_H^i > 0$ and $\sigma_N^i > 0$ are the coefficients of relative risk aversion pertaining to non-durable consumption, land services and leisure, respectively. Households’ different impatience is captured by the assumption that $\beta^P > \beta^I$. 

This ensures that, in steady state, patient and impatient households act as lenders and borrowers, respectively; cf. Woodford (1986). Utility maximization is subject to the sequence of budget constraints

\[ C_t^i + Q_t (H_t^i - H_{t-1}^i) + R_{t-1} B_{t-1}^i = B_t^i + W_t^i N_t^i, \]  

(3)

where \( B_t^i \) is the stock of one-period debt held at the end of period \( t \), \( R_t \) is the gross real interest rate on debt, \( Q_t \) is the price of land in units of consumption goods, and \( W_t^i \) is the real wage. Moreover, impatient households are subject to a collateral constraint on borrowing:

\[ B_t^i \leq s_t \frac{E_t \{Q_{t+1}^i\} H_t^i}{R_t}, \]  

(4)

which states that the maximum borrowable resources equal a fraction of the expected present value of durable goods holdings at the end of period \( t \). This constraint can be rationalized in terms of limited enforcement; cf. Kiyotaki and Moore (1997) and Iacoviello (2005). Lenders are assumed to pay a cost \( (1 - s_t) E_t \{Q_{t+1}^i\} H_t^i \) in period \( t + 1 \) if they are to repossess the collateral in case of default; hence, they will not lend more than \( s_t E_t \{Q_{t+1}^i\} H_t^i / R_t \) in period \( t \).

The term \( s_t \), the loan-to-value ratio, is assumed to satisfy

\[ \log s_t = \log s + \rho_s (\log s_{t-1} - \log s) + v_t, \quad 0 < \rho_s < 1, \]  

(5)

where \( v_t \sim N(0, \sigma_v^2) \). As our aim is to examine the implications of institutional changes in credit conditions, including regulatory measures, we interpret \( s \), which denotes the steady-state loan-to-value (LTV) ratio, as a proxy for the average stance of credit availability.

Patient households’ optimal behavior is described by the standard first-order conditions:

\[ (C_t^P)^{-\sigma_C^P} = \lambda_t^P, \]  

(6)

\[ \nu^P (1 - N_t^P)^{-\sigma_N^P} = \lambda_t^P W_t^P, \]  

(7)

\[ \lambda_t^P = \beta^P R_t E_t \{\lambda_{t+1}^P\}, \]  

(8)
\[ Q_t = \varepsilon_t \left( \frac{H_t^P}{\lambda_t^P} \right)^{-\sigma_H^P} + \beta^P \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^P}{\lambda_t^P} Q_{t+1} \right\}, \]  
\quad (9)

where \( \lambda_t^P \) is the multiplier associated with (3) for \( i = P \). Similarly, optimal behavior of impatient households is described by

\[ (C_t^I)^{-\sigma_C^I} = \lambda_t^I, \]  
\quad (10)

\[ \nu^I (1 - N_t^I)^{-\sigma_N^I} = \lambda_t^I W_t^I, \]  
\quad (11)

\[ \lambda_t^I - \mu_t^I = \beta^I R_t \mathbb{E}_t \left\{ \lambda_{t+1}^I \right\}, \]  
\quad (12)

\[ Q_t = \varepsilon_t \left( \frac{H_t^I}{\lambda_t^I} \right)^{-\sigma_H^I} + \beta^I \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^I}{\lambda_t^I} Q_{t+1} \right\} + s_t \frac{\mu_t}{\lambda_t^I} \mathbb{E}_t \left\{ Q_{t+1} \right\}, \]  
\quad (13)

where \( \lambda_t^I \) is the multiplier associated with (3) for \( i = I \), and \( \mu_t^I \) is the multiplier associated with (4). Additionally, the complementary slackness condition

\[ \mu_t^I \left( B_t^I - s_t \frac{\mathbb{E}_t \left\{ Q_{t+1} \right\} H_t^I}{R_t} \right) = 0, \]  
\quad (14)

must hold along with \( \mu_t^I \geq 0 \) and (4).

### 2.2 Entrepreneurs and firms

The representative entrepreneur has preferences defined over non-durables only (cf. Iacoviello, 2005; Liu et al., 2013), and maximizes

\[ E_0 \left\{ \sum_{t=0}^{\infty} \left( \beta^E \right)^t \frac{1}{1 - \sigma_C^E} \left( C_t^E \right)^{1 - \sigma_C^E} \right\}, \quad \sigma_C^E > 0, \]  
\quad (15)

where \( C_t^E \) is non-durable consumption, and where we assume that \( \beta^E < \beta^P \). This ensures that the entrepreneurs are borrowers in the steady state. Their relevant sequences of budget constraints are

\[ C_t^E + I_t + Q_t \left( H_t^E - H_{t-1}^E \right) + R_{t-1} B_{t-1}^E = B_t^E + r_t^K K_{t-1} + r_t^H H_{t-1}^E, \]  
\quad (16)

where \( I_t \) denotes investment in physical capital, \( K_{t-1} \) is the physical capital stock rented to firms at the end of period \( t - 1 \), and \( H_{t-1}^E \) is the stock of land rented to firms. Finally,
$r_{t-1}^K$ and $r_{t-1}^H$ are the rental rates on capital and land, respectively. Capital accumulation is given by the law of motion

$$K_t = (1 - \delta) K_{t-1} + \left[ 1 - \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right) \right]^2 I_t, \quad 1 > \delta > 0, \quad \Omega > 0,$$

(17)

where we have assumed quadratic investment adjustment costs.

Like impatient households, entrepreneurs are credit constrained, and they borrow using capital and land as collateral:

$$B_{t}^E \leq s_t E_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\},$$

(18)

where $Q_{t}^K$ denotes the price of installed capital in consumption units. For simplicity, we assume that households and entrepreneurs are subject to common credit limits.5

The optimal behavior of the entrepreneurs is characterized by the first-order conditions

$$(C_t^E)^{-\sigma_t^E} = \lambda_t^E,$$

(19)

$$\lambda_t^E - \mu_t^E = \beta^E R_t E_t \left\{ \lambda_{t+1}^E \right\},$$

(20)

$$-\lambda_t^E + \psi_t^E \left[ 1 - \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \left( \frac{I_t}{I_{t-1}} - 1 \right) \right] = \beta^E R_t E_t \left\{ \psi_{t+1}^E \Omega \left( \frac{I_{t+1}}{I_t} \right)^2 \left( 1 - \frac{I_{t+1}}{I_t} \right) \right\},$$

(21)

$$\psi_t^E = \beta^E r_t^K E_t \left\{ \lambda_{t+1}^E \right\} + \beta^E (1 - \delta) E_t \left\{ \psi_{t+1}^E \right\} + \mu_t^E s_t \frac{E_t \left\{ Q_{t+1}^K \right\}}{R_t},$$

(22)

$$Q_t = \beta^E r_t^H E_t \left\{ \frac{\lambda_{t+1}^E}{\lambda_t^E} \right\} + \beta^E E_t \left\{ \frac{\lambda_{t+1}^E}{\lambda_t^E} Q_{t+1} \right\} + s_t \frac{\mu_t^E E_t \left\{ Q_{t+1} \right\}}{R_t},$$

(23)

where $\lambda_t^E$, $\mu_t^E$ and $\psi_t^E$ are the multipliers associated with (16), (18) and (17), respectively. Moreover,

$$\mu_t^E \left( B_{t}^E - s_t E_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\} \right) = 0,$$

(24)

5The ratios of loans to assets in Figure 1 do not suggest large differences between households and firms. In Iacoviello (2005), the LTV ratio faced by entrepreneurs (0.89) is much higher than that faced by impatient households (0.55), while the opposite is the case in Gerali et al. (2010), who set 0.35 for entrepreneurs and 0.7 for households. In sum, in lack of conclusive evidence that LTV ratios faced by firms are systematically higher or lower than those faced by households, we assume that they are equal.
holds along with $\mu_t^E \geq 0$ and (18). Finally, the definition of $Q_t^K$ implies that

$$Q_t^K = \psi_t^E / \lambda_t^E. \quad (25)$$

Firms operate competitively under a constant-returns-to-scale technology. They rent capital and land from entrepreneurs and hire labor from both types of households, so as to maximize profits. The production technology for output, $Y_t$, is given by

$$Y_t = \tilde{A}_t \left[ (N_t^P)^{\alpha} (N_t^I)^{1-\alpha} \right]^{\gamma} \left[ (H_{t-1}^E)^{\phi} K_{t-1}^{1-\phi} \right]^{1-\gamma}, \quad 0 < \alpha, \phi, \gamma < 1, \quad (26)$$

with total factor productivity $\tilde{A}_t$ evolving according to

$$\log \tilde{A}_t = \log A + \rho_A (\log \tilde{A}_{t-1} - \log A) + z_t, \quad 0 < \rho_A < 1, \quad (27)$$

where $A > 0$ is the steady-state value of $\tilde{A}_t$, and $z_t \sim N(0, \sigma_A^2)$. Optimal factor-demand relations follow from the first-order conditions: \(^6\)

$$\alpha \gamma Y_t / N_t^P = W_t^P,$$

$$(1 - \alpha) \gamma Y_t / N_t^I = W_t^I,$$

$$(1 - \gamma) (1 - \phi) E_t \{Y_{t+1}^E \} / K_t = r_t^K,$$

$$(1 - \gamma) \phi E_t \{Y_{t+1}^E \} / H_t^E = r_t^H.$$

### 3 Equilibrium

We consider a competitive equilibrium where the markets for labor, capital and land for production all clear. These conditions can be incorporated conveniently by rewriting the

\(^6\)The assumption of imperfect substitutability between labor types follows Iacoviello (2005) and Justini-ano et al. (2015), among others. Iacoviello and Neri (2010) note that perfect substitutability complicates the solution of their model substantially, but yields similar results.
firms’ first-order conditions to account for the different masses of supplied factors:

\[ \alpha \gamma nY_t / [(1 - n_I - n_E) N_t^P] = W_t^P, \]  
\[ (1 - \alpha) \gamma nY_t / (n_I N_t^I) = W_t^I, \]  
\[ (1 - \gamma) (1 - \phi) nE_t \{Y_{t+1} \} / (n_E K_t) = \nu_t^K, \]  
\[ (1 - \gamma) \phi nE_t \{Y_{t+1} \} / (n_E H_t^E) = \nu_t^H. \]  

(28) (29) (30) (31)

where \( n \equiv \left[ (1 - n_I - n_E)^{\alpha} (n_I)^{1-\alpha} \right]^{-1} n_E^{1-\gamma} \). Moreover, as the supply of land is held fixed at \( H \), equilibrium in the the market for land implies that

\[ H = (1 - n_I - n_E) H_t^P + n_I H_t^I + n_E H_t^E. \]  

(32)

Also, the economy-wide net financial position is zero, such that

\[ (1 - n_I - n_E) B_t^P + n_I B_t^I + n_E B_t^E = 0. \]  

(33)

Finally, the aggregate resource constraint can be written as

\[ nY_t = (1 - n_I - n_E) C_t^P + n_I C_t^I + n_E C_t^E + n_E I_t. \]  

(34)

An equilibrium is sequences of quantities and prices, \( \{Y_t, C_t^P, C_t^I, C_t^E, I_t, H_t^P, H_t^I, H_t^E, K_t, N_t^P, N_t^I, B_t^P, B_t^I, B_t^E, \}_{t=0}^{\infty} \) and \( \{\lambda_t^P, \lambda_t^I, \lambda_t^E, \mu_t^I, \mu_t^E, \psi_t^I, \psi_t^E, \nu_t^K, \nu_t^H, Q_t^K, Q_t, W_t^P, W_t^I, R_t \}_{t=0}^{\infty} \), respectively, which conditional on sequences of shocks \( \{A_t, \varepsilon_t, s_t\}_{t=0}^{\infty} \) and initial conditions, satisfy the optimality conditions [(6), (7), (8), (9), (10), (11), (12), (13), (19), (20), (21), (22), (23), and (25)], the budget and credit constraints [(3) for \( i = P, I, \) (4), and (18)], as well as the technological constraints and market-clearing conditions [(17), (26), (28), (29), (30), (31) (32), (33), and (34)].

3.1 Solution method

In Appendix B we detail the steady state of the model, while Appendix C describes the log-linearized version, which is solved numerically. Due to the assumptions about the discount factors, \( \beta^P < \beta^I \) and \( \beta^P < \beta^E \), both collateral constraints are binding in steady
state. This follows as (20) pins down the steady-state real interest rate as \( R = 1/\beta^P \). As impatient households and entrepreneurs all have a higher subjective real rate of interest, their consumption levels can only be equalized across time when their collateral constraints bind. However, the optimal level of debt of one or both agents may fall short of the credit limit when the model is not at its steady state (say, in case of a large favorable shock), in which case the collateral constraint will be non-binding. In other words, our model features occasionally non-binding constraints, and thus non-linearities.

To account for this, we explicitly treat the collateral constraints as inequalities, and include the complementary slackness conditions, (14) and (24), in the model. In practice, we follow the approach of Holden and Paetz (2012), who develop a solution method for log-linearized DSGE models featuring inequalities. Building on Laséen and Svensson (2011), the central idea is to introduce a set of “shadow price shocks”, which ensures that each of the slackness conditions is satisfied in each period. If the conditions are violated, the shadow price shocks take on values exactly large enough to make the bounded variables, i.e. the debt levels of credit-constrained agents, equal to their (temporarily) unconstrained value. If the borrowing constraints are already binding, the shadow price shocks are zero. To ensure compatibility with rational expectations, the shocks are added to the model as “news shocks” in the sense of Schmitt-Grohé and Uribe (2012), i.e., they are fully anticipated. For first-order perturbations, our solution method yields similar simulated moments as the approach of Guerrieri and Iacoviello (2014); see, Holden and Paetz (2012). We present the details of the solution method in Appendix D.

### 3.2 Parameterization

We interpret one period as a quarter. The model is calibrated so that, in the steady state, it broadly matches a set of “big ratios” for the US economy as reported, e.g., by Liu et al. (2013). These include the ratios of capital and housing to output, and the share of time devoted to labor. As we describe in the following, this implies a set of relatively standard parameter values.

We let the steady-state LTV ratios faced by households and entrepreneurs, \( s \), be in the range \([0.01, 0.9]\) and report standard deviations, correlation coefficients, etc. for 19 different values within this range.\(^7\) In this way, we obtain a comprehensive picture of

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\(^7\)We set the upper and lower bound of this range so that given the standard deviation of the process
the effects of different LTV ratios on the macroeconomy. Moreover, this approach allows us to uncover potential non-monotonicities and to explore the effects of credit constraints becoming non-binding more often under high average LTV ratios. When we report impulse responses, however, we do so only for two values of $s$. The first is a “high” LTV regime, where $s = 0.7$. The second, a “low” LTV regime, has $s = 0.35$. The calibration of the model is in line with $s = 0.7$, as the ratios reported by Liu et al. (2013) are computed for the sample 1987–2007. These values, however, are not markedly different when we set $s = 0.35$.

We assume that agents are of identical mass; $n^I = n^E = 1/3$. As noted, impatient households and entrepreneurs have a lower discount factor than patient households. We therefore set $\beta^I = \beta^E = 0.97$ and $\beta^P = 0.99$. As for the curvature parameters in the utility functions, we set $\sigma^i_0 = 1$ for $i = \{P, I, E\}$, and similarly $\sigma^i_H = 1$ for $i = \{P, I\}$, so that all households have log utility in land services and non-durable consumption. The Frisch elasticity of labor supply is given by the inverse of $\sigma^i_N$ times the steady-state ratio of leisure to work. Calibrating the latter to around 3 for both types of households, a Frisch elasticity of labor supply of 1/3 implies $\sigma^i_N = 9$, $i = \{P, I\}$. We use $\nu^i = 0.27$ for $i = \{P, I\}$, which implies, as desired, that patient households work about 1/4 of their time in steady state, and impatient households slightly more. We calibrate the model so as to obtain a steady-state ratio of residential land to output around 1.45, and of commercial land to output around 0.65, both at the annual level, following values reported by Liu et al. (2013). This requires a value of $\varepsilon = 0.085$.

Regarding the parameters describing the technology of the model, we set $\gamma = 0.7$, implying a non-labor share in the production function close to 1/3. We set the labor income share of patient households to $\alpha = 0.7$, which is in the range of available estimates. For instance, Iacoviello (2005) obtains an estimate of 0.64 by matching impulse responses from his model to those from a VAR, while Iacoviello and Neri (2010) find a value of 0.79 using Bayesian estimation. The parameter $\phi$, which multiplied by $(1 - \gamma)$ measures land’s share of inputs, is set to 0.13, somewhat higher than the estimated value from Liu et al.

for $s_t$, the actual LTV ratios stay inside the interval $[0,1]$ with 95 pct. probability. We consider negative LTV ratios to be uninteresting, and while LTV ratios above 1 may sometimes occur empirically, we find it hard to reconcile with the idea of limited contract enforcement which we follow in this paper.

These values are within the range of values typically used in the literature; e.g., Mendoza (2010) reports 0.2–0.3, Calza et al. (2013) use 0.6, Liu et al. (2013) report 0.75, while Justiniano et al. (2014) set a value of 0.8.
We assume a capital depreciation rate of $\delta = 0.035$. The implied annual ratio of capital to output is around $1.15$, as in Liu et al. (2013). As to the investment adjustment cost parameter, $\Omega$, the empirical estimates range from nearly to zero in Liu et al. (2013) to above $26$ in Christiano et al. (2010) in estimated general equilibrium models. We choose an intermediate value of $\Omega = 4$.

For the technology shock, we choose values similar to those applied in most of the real business cycle literature, $\rho_A = 0.97$ and $\sigma_A = 0.005$ (see., e.g., Mandelman et al., 2011). For the land demand shock, we set $\rho_{\varepsilon} = 0.96$ and $\sigma_{\varepsilon} = 0.06$, which is close to the values used by Iacoviello and Neri (2010) and Liu et al. (2013). Finally, for the credit limit shock, we set the persistence parameter $\rho_s = 0.98$, in line with estimated coefficients from univariate regressions of the LTV-series displayed in the Introduction. We then calibrate $\sigma_s$ to obtain a standard deviation of the process for $\log s_t - \log s$ of around $0.06$, as estimated by Liu et al. (2013). This implies $\sigma_s = 0.0119$. All parameter values are shown in Appendix E.

4 Business cycles and credit limits

To examine the business cycle properties of the model, we first focus on output volatility. According to Figure 2 this increases in the average LTV ratio, at least up to $s \approx 0.8$, a value that we have argued to be in line with a financially liberalized economy. In fact, this credit limit is close to the typical LTV ratios on mortgage loans in a number of advanced countries, as reported, e.g., by Calza et al. (2013). Initially, an increase in the LTV ratio implies that credit-constrained agents experience a larger increase in their borrowing capacity in the face of a given increase in asset prices, and vice versa. With binding credit constraints, this translates into larger fluctuations in debt-financed consumption and investment activities by impatient households and entrepreneurs. In other words, these agents fully exploit the changes in their borrowing capacity to satisfy their relative impatience. In turn, this implies an increase in output volatility, as well as a stronger comovement between credit and economic activity. We elaborate further on the latter aspect in subsection 4.5.

Observe that our focus on the possibility of non-binding borrowing constraints becomes particularly important when the average LTV ratio approaches the upper bound of its sup-
Figure 2: Standard deviation of output for different LTV ratios.

Note: Numbers are median values from 501 stochastic model simulations of 2000 periods. All time series used to produce business cycle statistics have been preliminarily HP-filtered, setting $\lambda = 1600$.

In fact, beyond $s \approx 0.8$ output volatility falls in the LTV, but remains substantially higher than for economies with low LTV ratios.\(^9\) This drop in volatility is due to an increase in the frequency of episodes in which financial constraints are slack. As illustrated in Figure 3, the entrepreneur finds himself unconstrained as much as 50% of the time at very high LTV ratios; impatient households, however, only experience this rarely. In contrast to models with always binding constraints, we thus portray looser credit conditions as causing higher borrowing ability with the same collateral, and also as increasing the probability of not facing any (binding) credit constraints. Under such circumstances collateralized borrowing no longer acts as an amplifier of shocks to the economy. Subsection 4.4 digs deeper into this point.

We now examine how the model’s different shocks are propagated under different credit limits. Figure 4 shows the response of output to each of the three shocks under the low and high LTV regime, respectively. To appreciate how the effect of the shocks may be amplified through endogenous collateral values, we also illustrate the effects of each shock in an alternative version of the model, in which both types of borrowers face a fixed credit

\(^9\)By contrast, Mendicino (2012) finds that the amplification of technology shocks through collateral constraints increases in the credit limit, but tends to vanish as the LTV ratio approaches one. This happens as fully efficient debt enforcement is possible in her simple Kiyotaki and Moore (1997) economy.
Figure 3: Frequency of episodes of non-binding constraints for each agent.
Note: See the notes to Figure 2.

limit equal to their steady-state level of debt.

4.1 Technology shocks and amplification

Figure 4 shows that endogenous credit limits are an important source of amplification of technology shocks, especially at high LTV ratios, as compared with the alternative economy where collateral effects are shut off.\(^\text{10}\) This result contrasts with recent business cycle literature that finds little or no amplification of shocks to technology through credit constraints, cf. Kocherlakota (2000), Cordoba and Ripoll (2004), Liu et al. (2013), \textit{inter alia}. The key mechanism behind their lack of amplification is that productivity shocks move future dividends and the riskless rate of interest in the same direction, so that the discounted value of the collateral assets exerts a muted impact on collateral constraints. Our set up mainly differs from these studies by featuring two different types of credit-constrained agents. Entrepreneurs invest in land and capital goods to be employed for production purposes, while impatient households benefit from the stream of utility accruing from land holdings. Therefore, we combine the complementarity between land and physical capital in the production function with the effect of strategic complementarities stemming from the demand for land from both types of constrained agents. This demand channel

\(^{10}\)See also Figures F.1 and F.2 in Appendix F, which show the response of six key variables to a technology shock under high and low average LTV ratios.
is influenced by credit conditions, and opens up the route to a relevant amplification of technology shocks.

To provide an understanding of this result, some analytical insights are helpful. Consider first patient households’ Euler equation for land (9), which is solved forward to yield:

$$\lambda_t^P Q_t = E_d \left\{ \sum_{i=0}^{\infty} (\beta^P)^i \varepsilon_{t+i} (H_{t+i}^P)^{-\sigma P} \right\} \equiv \Upsilon_t. \quad (35)$$

Now consider the case where there are no land demand shocks; e.g., \( \varepsilon_t = \varepsilon, \forall t \). Since land does not depreciate, \( H_t^P \) is effectively an “idealized durable” in the sense of Barsky et al. (2007). This means that the intertemporal elasticity of substitution in land demand is close to infinity. Any short-term movements in \( H_t^P \) affect the right-hand side of (35) relatively little, as \( \beta^P \) is close to one. Hence, we can make the approximation:

$$\lambda_t^P Q_t = \Upsilon_t \approx \Upsilon. \quad (36)$$
This condition shows that movements in land prices (in absence of land-demand shocks) mirror movements in $\lambda_t^P$, i.e., patient households’ marginal utility of consumption. Hence, by (6) and (36), changes in the credit conditions will only affect land-price dynamics to the extent they affect unconstrained households’ consumption dynamics:

$$Q_t \approx (C_t^P)^{\sigma_t^E} \gamma.$$

We then turn to impatient households, whose Euler equation for land demand, (13), is a standard downward-sloping curve, conditional on all other variables. It also follows from (13) that steady-state credit conditions affect impatient households’ land demand, as the expected future price of collateral in present value has greater impact on current demand the higher is $s$. This is also the case for the entrepreneurs, to whom we now turn. Firms’ equilibrium demand for land is given by (31), which equates the marginal productivity of land with the rental rate. Using this condition to eliminate the rental rate in entrepreneurs’ Euler equation (23) yields (for the case $n = n^E$):

$$Q_t = \beta^E E_t \left\{ \frac{\lambda_{t+1}^E}{\lambda_t^E} Q_{t+1} \right\} + \beta^E \left[ (1 - \gamma) \phi E_t \left\{ Y_{t+1} \right\} / H_t^E \right] E_t \left\{ \frac{\lambda_{t+1}^E}{\lambda_t^E} \right\} + \mu_t^E s_t \frac{E_t \{ Q_{t+1} \}}{\lambda_t^E R_t}.$$

This is a downward-sloping function, as the marginal product of land decreases in $H_t^E$.

Figure 5 sketches the market for land when hit by a positive technology shock, $A_t - A > 0$. The horizontal schedules represent (37), the patient households’ demand. The downward-sloping schedules to the left represent (38), the entrepreneurs’ demand. Finally, the downward-sloping schedules to the right represent (38) and (13) in combination, i.e., entrepreneurs’ and impatient households’ demand. The steady-state holdings of land are indicated by the points “a” (since the stock of land is fixed, patient households’ land holdings are $H - H_t^E - H_t^I$).

When a positive technology shock hits, it has a direct effect on the demand of entrepreneurs because the marginal product of land increases. This is depicted as a movement of demand curves to the right. The increase in production possibilities will increase patient households’ consumption, leading to an increase in land prices, cf. (37). This induces a relaxation of entrepreneurs’ credit constraints and allows them to demand more land. One can represent this by a movement to the points “b”, where land holdings by entrepreneurs
Figure 5: Land prices and land holdings by different agents following a positive technology shock. The solid line represents the steady state. The dashed line represents movements in entrepreneurs’ and firms’ land demand without taking impatient households into account. The dotted line represents the general equilibrium effects when impatient households are taken into account.

have increased slightly, such that the output increase is somewhat amplified the following period.

These arguments, however, ignore important “second-round” effects in our model: The land price increase will also relax impatient households’ credit constraints, allowing them to demand more land. The relaxation of both types of agents’ credit constraints involves a positive credit externality, which allows both types to further increase their land holdings. This is illustrated by the movement to the points “c”. In this equilibrium, entrepreneurs’ land holdings increase even more (and the associated higher consumption possibilities for patient households drive $Q_t$ further up). This increase in entrepreneurial land generates the amplification of the output response in the following period; cf. Figure 4. Importantly, as seen previously from the entrepreneurs’ and households’ demand schedules, the sensitivity of land demand to changes in credit conditions crucially depends on the LTV ratio. Hence, the higher $s$, the stronger the amplification of output induced by the simultaneous relaxation of both agents’ credit limits. This explains why the amplification of technology shocks is significantly larger under the high LTV ratio, as compared with the low LTV ratio.
4.2 Land-demand and credit-market shocks

We now turn to the output effects of the two other shocks in the economy.\textsuperscript{11} As shown in Figure 4, land demand shocks are also amplified in the presence of endogenous credit limits, and in particular when the average LTV ratio is relatively high. The reasons are broadly in line with those outlined for the technology shock: land demand shocks drive up land prices, relaxing impatient households’ and entrepreneurs’ credit constraints. The agents react by increasing their borrowing, and more so under high LTV ratios.\textsuperscript{12} Shocks to credit limits feature a similar transmission, though they have a direct impact on agents’ borrowing capacity, so that they induce an even greater amplification. Once credit limits are relaxed, constrained agents bid up the prices of land and capital, which in turn raises their borrowing capacity even further in the case of endogenous credit limits, giving rise to an endogenous amplification of the initial shock.

It is important to note that, compared with other sources of exogenous perturbation, land-demand shocks have a short-lived impact on aggregate activity under high LTV ratios. This contrasts with Liu \textit{et al.} (2013), who find that in the presence of competing demand for land by patient households and entrepreneurs, the collateral constraint faced by the latter provides a powerful amplification of land demand shocks onto output and investment. This is not the case in our setup, due to the presence of an additional type of constrained agents—impatient households. Following a land demand shock, these agents increase their stock of land in a very persistent manner. As a result, land is eventually diverted away from firms, and thus from productive use.\textsuperscript{13} We do therefore not observe any amplification of investment after a land demand shock. In fact, investment increases by less compared to the case where agents are facing fixed credit limits; cf. figure F.3.

4.3 Variance decomposition

With the above in mind, it should come as no surprise that land-demand shocks play a minor role for business cycles in our model as compared with technology shocks. Figure

\textsuperscript{11}The effects of these two shocks on six key variables are displayed in Figures F.3-F.6 in Appendix F.

\textsuperscript{12}In fact, in equilibrium the entrepreneurs increase their land holdings on impact, despite the fact that they are the only agents not directly affected by the shock.

\textsuperscript{13}In a recent paper, Pinter (2014) has questioned the empirical specification of Liu \textit{et al.} (2013). He shows that adding credit-constrained households to their model improves the empirical performance and corroborates our model result that land-demand shocks are relatively unimportant for output fluctuations.
6 shows the relative importance of different shocks to output volatility, using a simple measure of our three structural shocks’ relative contribution to the volatility of various aggregates. For a generic variable \( x \), we define the contribution of shock \( \xi \) to its variance as
\[
\mathcal{V}(x, \xi) \equiv \frac{\text{var}[x] - \text{var}[x]_{-\xi}}{3\text{var}[x] - \sum_{\xi} \text{var}[x]_{-\xi}}, \quad \xi = A, \varepsilon, s,
\]
where \( \text{var}[x]_{-\xi} \) is the unconditional variance of \( x \), when the structural shock \( \xi \) is turned off.\(^{14}\) Land demand shocks contribute little to the volatility of output, regardless of the level of the average LTV ratio.\(^{15}\) Technology shocks appear as the main drivers of real activity at relatively low values of the average LTV ratio. In addition, financial shocks play a significant role for the volatility of real quantities at higher values of the average LTV ratio, as they directly create volatility in the credit constraints that shape the behavior of agents.\(^{16}\)

### 4.4 Business cycle asymmetries

Our approach enables us to explore the asymmetric role played by credit constraints. For high average LTV ratios, episodes of non-binding credit constraints become more frequent. This not only leads to a drop in the volatility of output, as documented above, but also induces negative skewness in the business cycle; see Figure 7. In other words, economic expansions become smaller than recessions.\(^{17}\) This finding crucially depends on the presence of occasionally non-binding constraints, and on the fact that only shocks that have a positive impact on asset prices have the potential to make credit constraints non-binding. Figure 7 shows that output skewness is practically zero at low values of the average LTV ratio, becoming increasingly negative as credit limits increase and the collateral constraints become non-binding more often; cf. Figure 3. Eventually, downturns

---

\(^{14}\)We use this measure since our numerical solution method does not allow us to carry out a standard variance decomposition. However, note that for a linear model, the \( \mathcal{V}(x, \xi) \)'s are equal to the ratios found by conventional variance decompositions.

\(^{15}\)However, as Figure F.8 in Appendix F shows, demand shocks represent the main source of variation in the land price. This follows immediately by allowing for land demand shocks to affect the right-hand side of (37). The impact of these shocks on land prices will be significant and largely unrelated to credit limits, cf. Figure F.4 where land price dynamics are seen to be relatively independent of \( s \) and of whether credit limits are endogenous or not.

\(^{16}\)As expected on a priori grounds, these shocks are the predominant driver of debt fluctuations, no matter the average LTV value, cf. Figure F.8.

\(^{17}\)For empirical evidence of asymmetries in the U.S. business cycle see, e.g., Neftci (1984) and Morley and Piger (2012).
are substantially deeper than booms.

To gain a better understanding of such asymmetric business cycles, it is useful to study the propagation of “large” shocks, which have the potential to make the borrowing constraints non-binding. Figure 8 displays the response of output to a set of large, positive shocks, as well as the mirror image of the response to equally-sized negative shocks. Under a high average LTV ratio, a positive technological innovation implies that the borrowing constraint of the entrepreneur becomes non-binding on impact, and remains slack for six quarters. Impatient households remain constrained throughout. During the first six quarters, entrepreneurs essentially behave like consumption smoothers, in the sense that they optimally choose to borrow less than they are able to. This attenuates the expansionary effects of the technology shock on their demand for land. This, in turn, dampens the boom in aggregate economic activity, as shown in the upper right panel. On the other hand, a negative shock to technology does not lead to a relaxation of either of the borrowing constraints, which therefore remain binding throughout. As a result, impatient households and entrepreneurs are forced to cut back on their borrowing in response to the drop in the value of their collateral assets. This produces a stronger response of output.

\[18\]

In the stochastic simulations, persistent combinations of positive “normal” shocks will be sufficient to make the constraints non-binding.
In other words, a large, negative technology shock has a larger impact on output than a similar-sized positive shock when occasionally non-binding constraints are taken into account. As already mentioned, this feature is more prevalent under high LTV ratios, giving rise to larger asymmetries in this case.\footnote{Note that this potential asymmetric effect of shocks was already pointed out by Kocherlakota (2000) in a simple model with full collateralization of assets.}

The second row of Figure 8 shows that a similar picture emerges for large land demand shocks. In this case, the shock makes the entrepreneurs’ collateral constraint non-binding during the first 14 quarters after the shock in the high LTV regime, while impatient households remain constrained throughout. As a result, entrepreneurs have no incentive to expand their borrowing capacity by increasing their stock of land. In fact, entrepreneurs lower their land holdings on impact, allowing patient and impatient households to increase their land holdings at the expense of non-durable consumption, which drops. The drop in land available for production leads to a drop in output, as seen from the right panel in the second row of the figure. On the other hand, there is no attenuation of large negative shocks to the economy. In that case, both collateral constraints remain binding, giving rise to a large drop in output. As for technology shocks, the skewness emerging from large demand shocks is much weaker when the LTV ratio is low, as seen from the left panel in the second row. In this case, the collateral constraint of the entrepreneur becomes non-binding.
Figure 8: Impulse responses of output to large (20 standard deviations) shocks to technology (row 1), land demand (row 2), and credit limits (row 3) for two different LTV ratios.

Notes: Light-grey periods are ones where the entrepreneurs are unconstrained; solid-grey periods are ones where all agents are unconstrained.

for only five quarters, while the impatient household again remains constrained.

The bottom row of Figure 8 shows the effects of large credit limit shocks. Under high LTV ratios, the entrepreneurs are unconstrained during the first 14 periods after a positive shock, while impatient households become unconstrained for one period. For the reasons discussed above, this leads to a muted response of output. In contrast, a large negative shock forces entrepreneurs into a sizeable reduction of their stock of land, and thus of the amount of land available for production. At the same time, impatient households are also forced to bring down their stock of land, which further depresses the land price, and thus the borrowing capacity of both constrained agents. The result is a large drop in output. For low LTV ratios, credit constraints remain binding throughout.
4.5 Comovement between debt and output

The increase in output volatility observed over most of the range of LTV ratios is, as argued above, a result of the enhanced ability of impatient households and entrepreneurs to engage in debt-financed consumption and investment. This also has implications for the comovement between credit extension and real economic activity. Figure 9 plots the correlation between aggregate debt and output. The correlation is increasing for LTV ratios up to $s \approx 0.75$, eventually inverting its pattern as the average LTV ratio approaches its upper bound and credit constraints become non-binding more frequently.

To understand what determines this pattern, it is useful to study how each type of agent makes consumption and investment decisions under different credit limits. Starting with the entrepreneurs, Figure 10 shows that the correlation between entrepreneurial consumption and debt increases up to $s \approx 0.6$, declining thereafter. At relatively low values of the average LTV ratio, a marginal relaxation of the collateral constraint allows this type of agent to increase its consumption by accessing more credit. However, as the average LTV ratio increases beyond a certain level, entrepreneurs find themselves unconstrained more and more often. This reduces the correlation between entrepreneurial consumption and debt. Similarly, the correlation between debt and investment in capital goods increases over most of the support for the credit limit, and declines when equity requirements be-
come loose enough. At high levels of the average LTV ratio, occasionally non-binding constraints are crucial in delinking debt and investment dynamics.

Like the entrepreneurs, impatient households also exploit higher credit limits to increase debt-financed consumption. However, we have seen that impatient households rarely find themselves financially unconstrained even at high LTV ratios. As a result, and unlike the entrepreneurs, the correlation between consumption and debt of impatient households increases monotonically with higher credit limits, cf. Figure 10. It is worth noting that impatient households behave in accordance with the “financial labor supply accelerator” channel of Campbell and Hercowitz (2009, 2011), according to which positive comovement between debt and labor is most prevalent under low LTV ratios. Tight credit limits force households to work longer hours in order to increase their spending in response to a positive shock. This property is also present in our setting, where looser credit conditions weaken the comovement between $B_{it}^d$ and $N_{it}^d$; cf. Figure 10. In our model with multiple agents and input factors, however, this effect does not translate into a weakened comovement between output and aggregate debt.\footnote{In the model of Campbell and Hercowitz (2009, 2011), labor of credit-constrained households equals output, so the effect also implies a drop in the comovement between output and debt when credit conditions}

Figure 10: Correlation coefficients between key variables for different LTV ratios.
Notes: See the notes to Figure 2.
The intertemporal choices of patient households mirror those of their credit-constrained counterparts in the debt market. Indeed, the correlation between their consumption and savings (i.e., aggregate credit) declines over most of the range of average LTV ratios. This indicates that patient households are increasingly willing to postpone consumption, while lending their available resources, at least up to \( s \approx 0.6 \). Beyond this point, as down-payment requirements drop further and credit constraints are relaxed, borrowers’ demand for additional credit slows down, and so does the propensity of patient households to save. This reverses the correlation pattern.

To sum up, greater credit availability initially allows financially constrained agents to give vent to their relatively higher impatience, resulting in a marked increase in the comovement of real activity and debt, which is only reverted when the economy is flooded with credit availability and instances of collateral constraints being non-binding become more frequent. It should be noted that these findings are in contrast to theories that advocate the role of financial liberalization in promoting a lower procyclicality of credit extension and smoother business cycles. According to Den Haan and Sterk (2010), the correlation between real mortgage debt and real GDP in the US dropped from 0.76 during the period 1954–1978 to 0.32 in the years 1984–2008.\(^{21}\) Several authors have proposed financial liberalization as one potential explanation for this finding, and a number of frameworks have been put forward to examine the influence of varying degrees of equity requirements on aggregate volatility and the comovement between real activity and debt.\(^{22}\) The results we have presented do not lend much support to this view.

5 Concluding comments

We have shown that, in a fairly conventional DSGE model with heterogeneous agents and multiple credit constraints, looser credit conditions initially generate increasing business cycle volatility. This pattern reverses at LTV ratios not far from those currently observed become more lax.

\(^{21}\)Campbell and Hercowitz (2011) show that the connection between labor and debt has weakened after the financial reforms of the 1980s. Iacoviello and Pavan (2012) present similar results for the correlation of real mortgage debt with real aggregate consumption, which dropped from 0.72 in the period 1952–1982 to 0.37 in the years 1983–2010.

\(^{22}\)See the critical review by Den Haan and Sterk (2010), who report a number of contributions to this strand of the literature, as well as various citations from policymakers and academics that support this view.
in many advanced economies. The non-monotonic relationship between the LTV ratio and macroeconomic volatility is indeed intertwined with the possibility of credit constraints becoming non-binding the higher are credit limits, which in itself paves the way for asymmetric business cycles.

As much macroprudential policy debate has been focusing on ways of imposing caps on credit availability, such a non-monotonicity questions the adequacy of these proposals. Also, a policy of countercyclical credit availability could produce adverse effects, if implemented at an average LTV ratio that is to the right of the maximum-volatility value. As our model is necessarily stylized, quantitative conclusions regarding the value of the LTV ratio at which output volatility reaches its peak should be drawn with care. Nonetheless, macroprudential regulators should consider that curbing LTV ratios might actually increase macroeconomic volatility in a situation characterized by lax equity requirements. In this case, tighter credit limits will increase the instances of binding collateral constraints, exposing borrowers to fluctuations in the availability of credit. In fact, using an estimated DSGE model, Guerrieri and Iacoviello (2014) demonstrate that non-binding credit constraints for households have played a role in the US housing boom leading up to the Great Recession, while the subsequent credit tightening has been crucial in deepening the contraction. This demonstrates that the macroprudential trade-off we point out is not merely a theoretical possibility.

Our results may also be inferred as general arguments pertaining to financial liberalization measures, but we have deliberately been silent on welfare implications, while focusing on the cyclical implications of different LTV ratios. However, our model features substantial level effects of increasing the LTV ratio: steady-state output in our calibration is 5.8% higher in the high LTV regime, compared with the low LTV regime. Therefore, a welfare evaluation will ultimately involve weighing level gains against volatility losses. For this purpose, our model may be too simple and stacked in favor of level gains, even though it features potential welfare losses of negative skewness that are rarely examined in business cycles analyses. A richer model with other realistic frictions is thus needed for a proper evaluation, and is left as a subject for future research.
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Appendices

A Liabilities and assets in the US

Figure 1 shows the ratio of liabilities to assets for households and firms in the United States, respectively. All data are taken from FRED (Federal Reserve Economic Data), Federal Reserve Bank of St. Louis. The primary source is Flow of Funds data from the Board of Governors of the Federal Reserve System. For business liabilities we use credit market instruments of nonfinancial corporate and noncorporate business. As assets we follow Liu et al. (2013) and use data on both sectors’ equipment and software as well as real estate at market value. For households and nonprofit organizations, we use credit market instruments as data for liabilities and use as assets both groups’ real estate at market value and equipment and software of nonprofit organizations. Further details are available upon request.

The ratios reported in Figure 1 are aggregate measures, and may therefore not reflect actual loan-to-value (LTV) requirements for the marginal borrower. Nonetheless, we report these figures since the flow of funds data delivers a continuous measure of LTV ratios covering the entire period 1949–2014. For households, the aggregate ratio of credit to assets in the economy is likely to underestimate the actual down-payment requirements faced by households applying for a mortgage loan, since loans and assets are not uniformly distributed across households. In our model, we distinguish between patient and impatient households, and we assume that only the latter group is faced with a collateral constraint. In the data, we have not made this distinction, so that the LTV ratio for households reported in figure 1 represents an average of the LTV of patient households (savers), who are likely to have many assets and small loans, and that of impatient households (borrowers), who on average have larger loans and fewer assets. Justiniano et al. (2014) use the Survey of Consumer Finances to make this distinction, and identify borrowers as households with liquid assets of a value less than two months of their income. Based on the surveys from 1992, 1995, and 1998, they arrive at an average LTV ratio for this group of around 0.8, while our measure fluctuates around 0.5 during the 1990’s. Another approach, following Duca et al. (2011), is to focus on first-time home-buyers, who are likely to fully exploit their borrowing capacity. Using data from the American Housing Survey, Duca et al. report LTV ratios approaching 0.9 towards the end of the 1990’s; reaching a peak of almost 0.95 before the onset of the recent crisis. While these alternative approaches are thus likely to result in higher...
levels of LTV ratios, we are interested in the development over time of these ratios. While we believe the Flow of Funds data provide the most comprehensive and consistent time series evidence in this respect, substantial increases over time in LTV ratios faced by households have been extensively documented; see, e.g., Campbell and Hercowitz (2009), Duca et al. (2011), Favilukis et al. (2013), and Boz and Mendoza (2014). It should be noted that for households, various government-sponsored programs directed at lowering the down-payment requirements faced by low-income or first-time home buyers have been enacted by different administrations (Chambers et al., 2009). These are likely to have contributed to the increase in the ratio of loans to assets illustrated in the left panel of Figure 1.

Likewise, the aggregate ratio of business loans to assets in the data may cover for a disparate distribution of credit and assets across firms. In general, the borrowing patterns and conditions of firms are more difficult to characterize than those of households, as their credit demand is more volatile, and their assets less uniform and often more difficult to assess. Liu et al. (2013) also use Flow of Funds data to calibrate the LTV ratio of entrepreneurs, and arrive at a value of 0.75. This ratio is based on an assumption that commercial real estate enters with a weight of 0.5 in the asset composition of firms. In contrast, the ratio we report in Figure 1 assigns a weight of 1 to commercial real estate. While the transformation of Liu et al. (2013) would result in higher LTV ratios at any point in time, it would not affect the finding of rising LTV ratios over time. The secular increase in firm leverage over the second half of the 20th century has also been documented by Graham et al. (2014) using data from the Compustat database. These authors report loan-to-asset ratios that are broadly in line with those we present. More generally, an enhanced access of firms to credit markets over time has been extensively documented in the literature. This involves, for instance, the emergence of a market for high-risk, high-yield bonds (Gertler and Lown, 1999), increased flexibility in firms’ financing decisions, and the resulting immoderation in financial quantities (Jermann and Quadrini, 2009).

B The steady state

The deterministic steady state of our model is described in the following, where variables without time subscripts are the steady-state values. We first consider the implications of the patient households’ optimality conditions. From (6) and (7), we get

\[ (C^P)^{\sigma^P C} = \lambda^P, \quad (B.1) \]

and

\[ \nu^P (1 - N^P)^{\sigma^KN} = \lambda^PW^P, \quad (B.2) \]

respectively. The steady-state gross interest rate on loans is recovered from (8):

\[ \beta^P R^P = \lambda^P, \quad R = \frac{1}{\beta^P}, \quad (B.3) \]

---

23It should be mentioned that they also show a Flow of Funds-based measure of debt to total assets at historical cost (or book value) for firms. The increase over time in this measure is smaller. However, we believe that the ratio of debt to pledgeable assets at market values (as shown in Figure 1) is the relevant measure for firms’ access to collateralized loans, and hence more appropriate for our purposes.

24We emphasize that Figure 1 reports a gross measure of firm leverage. Bates et al. (2009) report that firm leverage net of cash holdings has been declining since 1980, but that this decline is entirely due to a large increase in cash holdings.
emphasizing that it is the time preferences of the most patient individual that determine the steady-state rate of interest. From (9) we find

$$
\varepsilon \left( H^P \right)^{-\sigma_H^P} + \beta^P \lambda^P Q = \lambda^P Q, \\
\left( H^P \right)^{\sigma_H^P} = \frac{\varepsilon}{Q \lambda^P (1 - \beta^P)}. 
$$

(B.4)

Turning to the impatient households, (10) and (11), leads to

$$
(C^I)^{-\sigma_I^C} = \lambda^I, 
$$

(B.5)

and

$$
\nu^I \left( 1 - N^I \right)^{-\sigma_I^P} = \lambda^I W^I, 
$$

(B.6)

respectively. From (12) we obtain the steady-state value of the multiplier on the credit constraint:

$$
\mu^I = \lambda^I \left( 1 - \beta^I R \right), 
$$

which by use of (B.3) yields

$$
\mu^I = \lambda^I \left( 1 - \frac{\beta^I}{\beta^P} \right). 
$$

(B.7)

From (B.7), we see that in steady state $\mu^I > 0$ since $\beta^P > \beta^I$, which proves that the credit constraint (4) is binding in steady state. In a similar fashion, we get from (20):

$$
\mu^E = \lambda^E \left( 1 - \frac{\beta^E}{\beta^P} \right). 
$$

(B.8)

Hence, $\mu^E > 0$ implying that the entrepreneurs’ credit constraints, (18), are also binding in steady state. From (13) we get

$$
\varepsilon \left( H^I \right)^{-\sigma_H^I} + \beta^I \lambda^I Q + \mu^I s Q R = \lambda^I Q, \\
\left( H^I \right)^{\sigma_H^I} = \frac{\varepsilon}{Q \lambda^I \left[ 1 - \beta^I - \frac{\mu^I}{s} \frac{1}{R} \right]}, \\
\left( H^I \right)^{\sigma_I^H} = \frac{\varepsilon}{Q \lambda^I \left[ 1 - \beta^I - \left( 1 - \frac{\beta^I}{\beta^P} \right) s \beta^P \right]}, \\
\left( H^I \right)^{\sigma_I^H} = \frac{\varepsilon}{Q \lambda^I \left[ 1 - \beta^I - \left( \beta^P - \beta^I \right) s \right]}, 
$$

(B.9)

where the next-to-last line makes use of (B.3) and (B.7).

Turning to the remaining optimality conditions of the entrepreneurs, (19) gives

$$
(C^E)^{-\sigma_E^C} = \lambda^E, 
$$

(B.10)

and (21) implies

$$
\psi^E \left[ 1 - \frac{\Omega}{2} \left( \frac{I}{T} - 1 \right)^2 \right] - \psi^E \Omega^I \frac{I}{T} \left( \frac{I}{T} - 1 \right) + \beta^E \psi^E \Omega^I \left( \frac{I}{T} \right)^2 \left( \frac{I}{T} - 1 \right) = \lambda^E 
$$
leading to

$$\psi^E = \lambda^E.$$  \hspace{1cm} (B.11)

This reflects that there are no investment adjustment costs in steady state in our no-growth model. Therefore, the shadow value of a unit of capital equals the shadow value of wealth. Combining this with (25), we readily obtain

$$Q^K = 1.$$  \hspace{1cm} (B.12)

From (22) we obtain

$$\beta^E r^K \lambda^E + \beta^E (1 - \delta) \psi^E + \mu^E_s Q^K R^E = \psi^E,$$

$$\beta^E \lambda^E r^K + \beta^E (1 - \delta) \psi^E + \lambda^E \left(1 - \frac{\beta^E}{\beta^P}\right) s Q^K R^E = \psi^E,$$

$$1 + r^K - \delta = \frac{1 - (\beta^P - \beta^E) s Q^K}{\beta^E},$$  \hspace{1cm} (B.13)

where the second line uses (B.8), and the last uses (B.3) and (B.11), respectively. From (23) we find:

$$r^H = \frac{(1 - \beta^E) Q}{\beta^E} - \frac{\mu^E_s Q}{\lambda^E \beta^E R^E}.$$  \hspace{1cm} (B.14)

We then turn to the remaining equilibrium conditions in steady state. As we saw above, the two credit constraints are binding in steady state. Hence,

$$B^I = \frac{s Q H^I}{R},$$  \hspace{1cm} (B.15)

$$B^E = \frac{s Q^K K + Q H^E}{R},$$  \hspace{1cm} (B.16)

The production function is

$$Y = \left[(N^P)^\alpha (N^I)^{1-\alpha}\right]^{\gamma} \left[(H^E)^\phi K^{1-\phi}\right]^{1-\gamma}.$$  \hspace{1cm} (B.17)

The steady-state versions of the firms’ first-order conditions taking market clearing conditions into account, (28)–(31), are

$$\alpha^\gamma \frac{nY}{(1 - nI - nE) N^P} = W^P,$$  \hspace{1cm} (B.18)

$$(1 - \alpha)^\gamma \frac{nY}{nI N^I} = \phi,$$  \hspace{1cm} (B.19)

$$\frac{(1 - \gamma) (1 - \phi) nY}{nE K} = \phi,$$  \hspace{1cm} (B.20)

$$\frac{(1 - \gamma) \phi nY}{nE H^E} = \phi.$$  \hspace{1cm} (B.21)

In steady state, the law of motion for capital implies

$$I = \delta K.$$  \hspace{1cm} (B.22)
We have the following steady-state resource constraints:
\[ nY = (1 - n_I - n_E) C^P + n_I C^I + n_E C^E + n_E I, \]  
\[ H = (1 - n_I - n_E) H^P + n_I H^I + n_E H^E, \]  
\[ (1 - n_I - n_E) B^P + n_I B^I + n_E B^E = 0. \]  
(B.23)  
(B.24)  
(B.25)

Also, we have the steady-state versions of the agents’ budget constraints:
\[ C^P = W^P N^P - (R - 1) B^P, \]  
\[ C^I = W^I N^I - (R - 1) B^I, \]  
\[ C^E + I = r^K K + r^H H^E - (R - 1) B^E \]  
(B.26)  
(B.27)  
(B.28)

(One of these is redundant by Walras’ law.)

We therefore have that the steady state is characterized by the vector
\[ \begin{bmatrix} Y, C^P, C^I, C^E, I, H^P, H^I, H^E, K, N^P, N^I, B^P, B^I, B^E, \\ Q, Q^K, R, r^K, r^H, W^P, W^I, \lambda^P, \lambda^I, \lambda^E, \mu^I, \mu^E, \psi^E \end{bmatrix}. \]

These 27 variables are determined by the 27 equations (B.1), (B.2), (B.3), (B.4), (B.5), (B.6), (B.7), (B.8), (B.9), (B.10), (B.11), (B.13), (B.14), (B.15), (B.16), (B.17), (B.18), (B.19), (B.20), (B.21), (B.22), (B.23), (B.24), (B.25), (B.26), (B.27), and (B.12).

We now briefly proceed with a characterization of the steady state, wherein we compute some variables in ratios to output in closed form. Then we reduce the system to one of seven equations in central quantities, which is solved numerically, conditional on these ratios. The remaining 19 variables then follow explicitly from the characterizations given above. First, combine (B.20) and (B.13) to get an expression for capital-output ratio:
\[ \frac{n_E K}{nY} = \frac{\beta^E (1 - \gamma) (1 - \phi)}{1 - (\beta^P - \beta^E) s - \beta^E (1 - \delta)}. \]  
(B.29)

where we have used that \(Q^K = 1\) by (B.12), Then we combine (B.14) and (B.21) to get an expression for entrepreneurs’ housing-output ratio:
\[ \frac{(1 - \gamma) \phi nY}{n_E H^E} = \frac{(1 - \beta^E) Q - \mu^E s Q}{\lambda^E \beta^E R}; \]  
\[ \frac{nY}{n_E H^E} = \frac{(1 - \beta^E) Q \lambda^E R - \mu^E s Q}{(1 - \gamma) \phi \beta^E \lambda^E R}, \]  
\[ \frac{Q n_E H^E}{nY} = \frac{(1 - \gamma) \phi \beta^E}{(1 - \beta^E) - (\beta^P - \beta^E) s}, \]  
(B.30)

where the last line uses (B.8). Again using that \(Q^K = 1\), the borrowing constraint for entrepreneurs (B.16) can be rewritten in terms of ratios to output as
\[ \frac{n_E B^E}{nY} = \frac{s}{R} \left( \frac{n_E K}{nY} + \frac{Q n_E H^E}{nY} \right), \]
which by use of (B.29), (B.30) and (B.3) imply
\[ \frac{n_E B^E}{nY} = \beta^P \left( \frac{\beta^E (1 - \gamma) (1 - \phi)}{1 - (\beta^P - \beta^E) s - \beta^E (1 - \delta)} + \frac{(1 - \gamma) \phi \beta^E}{(1 - \beta^E) - (\beta^P - \beta^E) s} \right). \]  
(B.31)
This closed-form solution of the entrepreneurs’ steady-state loan-to-output ratio is central in setting up a subsystem of seven central variables. First, it can be used with the entrepreneur’s budget constraint, \((B.28)\), in ratio to output:

\[
\frac{n_E C^E}{nY} + \frac{n_E I}{nY} = r^K \frac{n_E K}{nY} + r^H \frac{n_E H^E}{nY} - (R - 1) \frac{n_E B^E}{nY},
\]

which by use of \((B.22)\) becomes

\[
\frac{n_E C^E}{nY} = (r^K - \delta) \frac{n_E K}{nY} + r^H \frac{n_E H^E}{nY} - (R - 1) \frac{n_E B^E}{nY}.
\]

Using \((B.13)\) and \((B.21)\) we get

\[
\frac{n_E C^E}{nY} = \left(1 - \beta^E - (\beta^P - \beta^E) \frac{s}{\beta^E}\right) \frac{n_E K}{nY} + (1 - \gamma) \phi \frac{n_E H^E}{nY} - (R - 1) \frac{n_E B^E}{nY},
\]

which by use of \((B.29)\) provides the entrepreneurs’ consumption-to-output ratio:

\[
\frac{n_E C^E}{nY} = \left(1 - \gamma\right) \left(1 - \phi\right) \left[1 - \beta^E - (\beta^P - \beta^E) \frac{s}{\beta^E}\right] + (1 - \gamma) \phi - \frac{1 - \beta^P}{\beta^P} \frac{n_E B^E}{nY},
\] (B.32)

Then turn to the impatient households. In ratio to output, their budget constraints are, cf. \((B.27)\),

\[
\frac{n_I C^I}{nY} = \frac{n_I W^I N^I}{nY} - (R - 1) \frac{n_I B^I}{nY},
\]

which by use of \((B.19)\) and \((B.3)\) becomes

\[
\frac{n_I C^I}{nY} = (1 - \alpha) \gamma - \frac{1 - \beta^P}{\beta^P} \frac{n_I B^I}{nY}.
\]

Likewise, the patient households’ budget constraints are written as, cf. \((B.26)\),

\[
\frac{(1 - n_I - n_E) C^P}{nY} = \frac{(1 - n_I - n_E) W^P N^P}{nY} - (R - 1) \frac{(1 - n_I - n_E) B^P}{nY},
\]

which by use of \((B.18)\) and \((B.3)\) becomes

\[
\frac{(1 - n_I - n_E) C^P}{nY} = \alpha \gamma - \frac{1 - \beta^P}{\beta^P} \frac{(1 - n_I - n_E) B^P}{nY}.
\]

Adding these constraints gives

\[
\frac{n_I C^I + (1 - n_I - n_E) C^P}{nY} = \gamma + \frac{1 - \beta^P}{\beta^P} \frac{n_E B^E}{nY},
\] (B.33)

where \((B.25)\) has been invoked. Note that the right-hand-side of \((B.33)\) is known by \((B.31)\).

Combining \((B.1)\), \((B.2)\) and \((B.18)\) gives the steady-state equilibrium condition for the labor market for impatient households:

\[
\nu^P \left(1 - N^P\right)^{-\sigma^E} \left(C^P\right)^{\sigma^E} = \alpha \gamma \frac{nY}{(1 - n_I - n_E) N^P}.
\] (B.34)

Similarly, \((B.5)\), \((B.6)\) and \((B.19)\) characterize the labor-market equilibrium for impatient house-
holds:
\[ \nu^I (1 - N^I)^{-\sigma^I_N} (C^I)^{\sigma^I_C} = (1 - \alpha) \gamma \frac{nY}{n_I N^I}. \]  
(B.35)

Combining the two households’ land demand expressions, (B.4) and (B.9), gives
\[ \frac{(H^I)^{\sigma^I_H}}{(H^P)^{\sigma^P_H}} = \frac{\lambda^P (1 - \beta^P)}{\lambda^I [1 - \beta^I - (\beta^P - \beta^I) s]}. \]
Eliminating the multipliers by (B.1) and (B.5), and eliminating \( H^P \) by (B.24), we obtain the following land-market equilibrium characterization:
\[ \frac{(H^I)^{\sigma^I_H} (1 - n^I - n^E)^{\sigma^P_H}}{(H - n^I H^I - n^E H^E)^{\sigma^P_H}} = \frac{(C^P)^{-\sigma^P_C} (1 - \beta^P)}{(C^I)^{-\sigma^I_C} [1 - \beta^I - (\beta^P - \beta^I) s]} \cdot \]
\[ \frac{(H^I)^{\sigma^I_H} (C^P)^{\sigma^P_C} (1 - n^I - n^E)^{\sigma^P_H}}{(H - n^I H^I - n^E H^E)^{\sigma^P_H} (C^I)^{\sigma^I_C}} = \frac{1 - \beta^P}{1 - \beta^I - (\beta^P - \beta^I) s}. \]  
(B.36)

We also take the impatient households’ borrowing constraint into consideration. Using (B.15) to eliminate \( B^I \) in the budget constraint, it becomes
\[ \frac{n_I C^I}{nY} = (1 - \alpha) \gamma - \frac{1 - \beta^P}{\beta^P} \frac{s Q n_I H^I}{nY R}, \]
\[ = (1 - \alpha) \gamma - (1 - \beta^P) \frac{s Q n_I H^I}{nY}. \]  
(B.37)

We can use that (B.9) implies
\[ (H^I)^{\sigma^I_H^{-1}} = \frac{\varepsilon}{Q H^I \lambda^I [1 - \beta^I - (\beta^P - \beta^I) s]} \]
and thus, again using (B.5),
\[ Q H^I = \frac{\varepsilon (H^I)^{1-\sigma^I_H} (C^I)^{\sigma^I_C}}{1 - \beta^I - (\beta^P - \beta^I) s}, \]  
(B.38)
\[ Q = \frac{\varepsilon (H^I)^{1-\sigma^I_H} (C^I)^{\sigma^I_C}}{1 - \beta^I - (\beta^P - \beta^I) s}. \]  
(B.39)

We then use (B.38) to rewrite the consumption-output ratio for impatient households (B.37) as:
\[ \frac{n_I C^I}{nY} = (1 - \alpha) \gamma - (1 - \beta^P) \frac{s Q n_I H^I}{nY} \]
\[ = (1 - \alpha) \gamma - (1 - \beta^P) \left( \frac{s Q n_I}{nY} \varepsilon (H^I)^{1-\sigma^I_H} (C^I)^{\sigma^I_C} \right) \]  
(B.40)

Likewise, we rewrite the entrepreneurs’ land to output ratio by using (B.39) to eliminate \( Q \) from (B.30):
\[ \frac{n_E H^E}{nY} = \frac{(1 - \gamma) \phi \beta^E}{1 - \beta^E - (\beta^P - \beta^I) s} \frac{1 - \beta^I - (\beta^P - \beta^I) s}{\varepsilon (H^I)^{1-\sigma^I_H} (C^I)^{\sigma^I_C}}. \]  
(B.41)
Finally, the production function (B.17) is rewritten as a function of the derived ratios:

\[
\begin{align*}
nY &= A \left\{ \left[ (1-n_E-n_I) N^p \right]^\alpha (n_I N^I)^{1-\alpha} \right\} \gamma \left[ \left( \frac{n_E H^E}{nY} \right)^{\phi} \left( \frac{n_E K}{nY} \right)^{1-\phi} \right]^{1-\gamma}, \\
nY &= A \left\{ \left[ (1-n_E-n_I) N^p \right]^\alpha (n_I N^I)^{1-\alpha} \right\} \gamma \left[ \left( \frac{n_E H^E}{nY} \right)^{\phi} \left( \frac{n_E K}{nY} \right)^{1-\phi} nY \right]^{1-\gamma}, \\
(nY)^\gamma &= A \left\{ \left[ (1-n_E-n_I) N^p \right]^\alpha (n_I N^I)^{1-\alpha} \right\} \gamma \left[ \left( \frac{n_E H^E}{nY} \right)^{\phi} \left( \frac{n_E K}{nY} \right)^{1-\phi} \right]^{1-\gamma}.
\end{align*}
\]

Using (B.29), we finally obtain

\[
nY = A^\frac{1}{\gamma} \left[ (1-n_E-n_I) N^p \right]^\alpha (n_I N^I)^{1-\alpha} \left[ \left( \frac{n_E H^E}{nY} \right)^{\phi} \left( \frac{\beta^E (1-\gamma) (1-\phi)}{1- (\beta^p - \beta^E) s - \beta^E (1-\delta)} \right) \right]^{1-\frac{1}{\gamma}}.
\]

We have now reduced the steady-state to a matter of finding the vector

\[
\]

which satisfies the equations (B.33), (B.34), (B.35), (B.36), (B.40), (B.41) and (B.42), given the solution for \( n_E B^E / (nY) \), (B.31), and given all parameters and exogenous variables of the model. We compute the vector numerically using \texttt{fsolve} in \texttt{MATLAB}. The remaining 19 variables then follow analytically from the steady-state equations presented above.

## C The log-linearized model

We log-linearize the model around the steady state found in the previous section. In the following, we let \( \tilde{X}_t \) denote the log-deviation of a generic variable \( X_t \) from its steady state value \( X \), except for the following variables. For the interest rates, \( \tilde{R}_t \equiv R_t - R, \tilde{r}_t^H \equiv r_t^H - r^H \) and \( \tilde{r}_t^K \equiv r_t^K - r^K \), and for debt, \( \tilde{B}_t^i \equiv n_i (B_t^i - B^i) / (nY), i = P, I, E \). We first derive the log-linear versions of the agents’ optimality conditions and conclude with the expressions for market clearing.

### C.1 Optimality conditions of the patient households

Equations (6), (7) and (8) readily becomes

\[
-\sigma C \tilde{C}_t^P = \tilde{\lambda}_t^P, \tag{C.1}
\]

\[
\sigma_N \frac{N^P}{1-N^P} \tilde{X}_t^P = \tilde{\lambda}_t^P + \tilde{W}_t^P, \tag{C.2}
\]

\[
\beta^P \tilde{R}_t + E_t \left\{ \tilde{\lambda}_{t+1}^P \right\} = \tilde{\lambda}_t^P, \tag{C.3}
\]

Log-linearization of (9) yields

\[
\varepsilon (H^P)^{-\sigma_H^P} \left( \tilde{v}_t - \sigma_H^P \tilde{H}_t^P \right) + \beta^P \lambda^P Q E_t \left\{ \tilde{\lambda}_{t+1}^P + \tilde{Q}_{t+1} \right\} = \lambda^P Q \left\{ \tilde{\lambda}_t^P + \tilde{Q}_t \right\}.
\]

Now use steady-state equation (B.4) to get

\[
-\sigma_H^P Q \lambda^P (1-\beta^P) \tilde{H}_t^P + Q \lambda^P (1-\beta^P) \tilde{v}_t + \beta^P \lambda^P Q E_t \left\{ \tilde{\lambda}_{t+1}^P + \tilde{Q}_{t+1} \right\} = \lambda^P Q \left\{ \tilde{\lambda}_t^P + \tilde{Q}_t \right\}.
\]
and thereby
\[ \beta^P E_t \left\{ \lambda_{t+1}^P + \hat{Q}_{t+1} \right\} - \sigma_H^P (1 - \beta^P) \hat{H}_t^P + (1 - \beta^P) \hat{\varepsilon}_t = \lambda_t^P + \hat{Q}_t. \]  

(C.4)

Moreover, the log-linearized budget constraint holds:
\[ \frac{(1 - n_I - n_E) C^P}{nY} \hat{C}_t^P + \frac{Q (1 - n_I - n_E) H^P}{nY} (\hat{H}_t^P - \hat{H}_{t-1}^P) + \frac{(1 - n_I - n_E) B^P}{nY} \hat{R}_{t-1} + \frac{1}{\beta^P} \hat{B}_t^P = \hat{B}_t^P + \alpha \gamma \left( \hat{W}_t^P + \hat{N}_t^P \right). \]

where we have used (B.18). This constraint, however, does not feature in our MATLAB codes (we use the impatient households’ and entrepreneurs’ budget constraint and the economy-wide resource constraint).

C.2 Optimality conditions of the impatient households

From (10), (11) and (12) we obtain
\[ -\sigma_C^I \hat{C}_t^I = \lambda_t^I, \]  

(C.5)

\[ \sigma_P^I \frac{N_t^I}{1 - N_t^I} \hat{N}_t^I = \lambda_t^I + \hat{W}_t^I, \]  

(C.6)

and
\[ \beta^I \lambda^I \hat{R}_t + \beta^I R \lambda^I E_t \left\{ \hat{\lambda}_{t+1}^I \right\} + \mu^I \hat{\mu}_t^I = \lambda^I \hat{\lambda}_t^I, \]

respectively. The last expression is rewritten by use of (B.7):
\[ \beta^I \hat{R}_t + \beta^I R E_t \left\{ \hat{\lambda}_{t+1}^I \right\} + \left( 1 - \frac{\beta^I}{\beta^P} \right) \hat{\mu}_t^I = \lambda^I \hat{\lambda}_t^I. \]  

(C.7)

Furthermore, (13) becomes
\[ \varepsilon (H^I)^{-\sigma_H^I} \left( \hat{\varepsilon}_t - \sigma_H^I \hat{H}_t^I \right) + \beta^I \lambda^I Q E_t \left\{ \hat{\lambda}_{t+1}^I + \hat{Q}_{t+1} \right\} + \mu^I \hat{\mu}_t^I + \hat{\varepsilon}_t + E_t \left\{ \hat{Q}_{t+1} \right\} - \beta^P \hat{R}_t \]
\[ = \lambda^I Q \left( \hat{\lambda}_t^I + \hat{Q}_t \right), \]

which by use of (B.7) and (B.9) becomes
\[ Q \lambda^I \left[ 1 - \beta^I - (\beta^P - \beta^I) s \right] \left( \hat{\varepsilon}_t - \sigma_H^I \hat{H}_t^I \right) + \beta^I \lambda^I Q E_t \left\{ \hat{\lambda}_{t+1}^I + \hat{Q}_{t+1} \right\} + \lambda^I \left( 1 - \frac{\beta^I}{\beta^P} \right) s Q \frac{R}{E} \left[ \hat{\mu}_t^I + \hat{\varepsilon}_t + E_t \left\{ \hat{Q}_{t+1} \right\} - \beta^P \hat{R}_t \right] \]
\[ = \lambda^I Q \left( \hat{\lambda}_t^I + \hat{Q}_t \right), \]
where we have again used (B.3). The budget constraint becomes

\[
\frac{n_tC^l}{nY} \hat{C}_t^l + \frac{Qn_H}{nY} \left( \hat{H}_t^l - \hat{H}_{t-1}^l \right) + \frac{n_B}{nY} \hat{R}_{t-1} + 1 \frac{1}{\beta^P} \hat{B}_{t-1}^l = \hat{B}_t^l + (1 - \alpha) \gamma \left( \bar{W}_t^l + \tilde{N}_t^l \right), \tag{C.9}
\]

where we have used (B.19). Finally, the log-linearized version of (4) holds:

\[
\frac{nY}{nB^l} \hat{B}_t^l \leq s_t^l + E_t \left\{ \hat{Q}_{t+1} \right\} + \hat{H}_t^l - \beta^P \hat{R}_t. \tag{C.10}
\]

Note that while the credit constraint binds in steady state, cf. (B.15), we allow it to be non-binding outside steady state.

### C.3 Optimality conditions of the entrepreneurs

From (B.8) and (20) we get

\[
-\sigma^E C_t^E = \hat{\lambda}_t^E, \tag{C.11}
\]

\[
\beta^E \hat{R}_t + \beta^E R E_t \left\{ \hat{\lambda}_{t+1}^E \right\} + \left( 1 - \frac{\beta^E}{\beta^P} \right) \hat{\mu}_t^E = \hat{\lambda}_t^E. \tag{C.12}
\]

From (21) we derive

\[
\hat{\psi}_t^E - \Omega \left( 1 + \beta^E \right) \hat{I}_t + \Omega \hat{I}_{t-1} + \beta^E \Omega E_t \left\{ \hat{I}_{t+1} \right\} = \hat{\lambda}_t^E, \tag{C.13}
\]

where we have made use of (B.11). Equation (22) becomes

\[
\beta^E \hat{K}_t^E + \beta^E r K E_t \left\{ \hat{\lambda}_{t+1}^E \right\} + \left( 1 - \delta \right) \beta^E E_t \left\{ \hat{\psi}_{t+1}^E \right\} + \left( \beta^P - \beta^E \right) sQ \left( \hat{\mu}_t^E + \hat{s}_t + E_t \left\{ \hat{Q}_{t+1}^K \right\} - \beta^P \hat{R}_t \right)
\]

\[
= \hat{\psi}_t^E \tag{C.14}
\]

where we have used (B.11) and (B.3). Moreover, (25) becomes

\[
\hat{\psi}_t^E = \hat{\lambda}_t^E + \hat{Q}_t^K. \tag{C.15}
\]

Finally, (23) is approximated as

\[
\beta^E Q \left( E_t \left\{ \hat{\lambda}_{t+1}^E \right\} + E_t \left\{ \hat{Q}_{t+1}^E \right\} \right) + \beta^E r H E_t \left\{ \hat{\lambda}_{t+1}^E \right\} + \left( \frac{1}{\beta^H} \hat{r}_t^H \right) + \left( \beta^P - \beta^E \right) sQ \left( \hat{\mu}_t^E + \hat{s}_t + E_t \left\{ \hat{Q}_{t+1}^E \right\} - \beta^P \hat{R}_t \right)
\]

\[
= Q \left( \hat{\lambda}_t^E + \hat{Q}_t \right). \tag{C.16}
\]
Furthermore, the budget constraint for entrepreneurs becomes

\[
\frac{n_E C^E}{n_Y} \hat{C}_t + \frac{n_E I}{n_Y} \hat{I}_t + \frac{Q n_E H^E}{n_Y} \left( \hat{H}_t^E - \hat{H}_{t-1}^E \right) + \frac{n_E B^E}{n_Y} \hat{R}_{t-1} + \frac{1}{\beta^E} \hat{B}_{t-1}^E = \hat{B}_t^E + \frac{n_E K}{n_Y} \hat{K}_{t-1}^E + \frac{n_E H^E}{n_Y} \hat{H}_{t-1}^E + (1 - \gamma) \phi \hat{H}_{t-1}^E + (1 - \gamma) (1 - \phi) \hat{K}_{t-1} .
\]  
(C.17)

where we have used (B.20) and (B.21). The borrowing constraint must be satisfied:

\[
\frac{n_Y}{n_E} \hat{B}_t^E \leq s \frac{(K + QH^E)}{R} \hat{s} - \frac{s}{R^2} (K + QH^E) \hat{R}_t + \frac{s K}{R} E_t \left\{ \hat{Q}_{t+1}^K \right\} 
+ \frac{s K}{R} \hat{K}_t + \frac{s QH^E}{R} E_t \left\{ \hat{Q}_{t+1} \right\} + \frac{s QH^E}{R} \hat{H}_t^E .
\]

Dividing by the steady-state values on both sides:

\[
\frac{n_Y}{n_E} \hat{B}_t^E \leq \hat{s} - \beta^E \hat{R}_t + \frac{K}{K + QH^E} E_t \left\{ \hat{Q}_{t+1}^K \right\} + \frac{QH^E}{K + QH^E} \hat{K}_t + \frac{QH^E}{K + QH^E} \hat{H}_t^E ,
\]

yielding

\[
\frac{n_Y}{n_E} \hat{B}_t^E \leq \hat{s} - \beta^E \hat{R}_t + \frac{K}{K + QH^E} E_t \left\{ \hat{Q}_{t+1}^K \right\} + \frac{QH^E}{K + QH^E} \hat{K}_t + \frac{QH^E}{K + QH^E} \hat{H}_t^E .
\]  
(C.18)

### C.4 Optimality conditions of the firms

The first-order conditions of firms, (28), (29), (30) and (31), are readily rewritten as

\[
\hat{Y}_t - \hat{N}_t^P = \hat{W}_t^P ,
\]  
(C.19)

\[
\hat{Y}_t - \hat{N}_t^I = \hat{W}_t^I ,
\]  
(C.20)

\[
E_t \left\{ \hat{Y}_{t+1} \right\} - \hat{K}_t = \left( \gamma^K \right)^{-1} \hat{r}_t^K ,
\]  
(C.21)

\[
E_t \left\{ \hat{Y}_{t+1} \right\} - \hat{H}_t = \left( \gamma^H \right)^{-1} \hat{r}_t^H ,
\]  
(C.22)

respectively.

### C.5 Market clearing and resource constraints

From the law of motion for capital, (17), we get:

\[
\hat{K}_t = (1 - \delta) \hat{K}_{t-1} + \delta \hat{I}_t .
\]  
(C.23)

where we have used (B.22). Moreover, from the resource constraint, (34), we have:

\[
\hat{Y}_t = \frac{(1 - n_t - n_E)}{n_Y} \hat{C}_t^P + \frac{n_t C^I}{n_Y} \hat{C}_t^I + \frac{n_E C^E}{n_Y} \hat{C}_t^E + \delta \frac{n_E K}{n_Y} \hat{I}_t .
\]  
(C.24)

We also have the linearized versions of (26), (32) and (33):

\[
\hat{Y}_t = \hat{A}_t + \alpha \gamma \hat{N}_t^P + (1 - \alpha) \gamma \hat{N}_t^I + (1 - \gamma) (1 - \phi) \hat{K}_{t-1} + (1 - \gamma) \phi \hat{H}_{t-1}^E .
\]  
(C.25)
0 = (1 - n_I - n_E) H^P \tilde{H}_t^P + n_I H^I \tilde{H}_t^I + n_E H^E \tilde{H}_t^E 
(C.26)
0 = \tilde{B}_t^P + \tilde{B}_t^I + \tilde{B}_t^E 
(C.27)

Finally, we have the shock processes. For the technology shock, we have from (27):
\[ \tilde{A}_t = \rho_A \tilde{A}_{t-1} + z_t. \] (C.28)

Furthermore, we have from (2):
\[ \tilde{\xi}_t = \rho_\xi \tilde{\xi}_{t-1} + u_t \] (C.29)

Finally, we have from (5) that
\[ \tilde{s}_t = \rho_s \tilde{s}_{t-1} + v_t, \] (C.30)

which completes our list of log-linearized equations.

The log-linearized system consists of 30 equations: 18 first-order conditions, 2 budget constraints, 2 credit constraints, 1 production function, 3 market clearing conditions, 1 capital accumulation equation, and 3 shock processes. The 30 variables of the system are given by the vector
\[
\begin{bmatrix}
\hat{C}_t^P, \hat{C}_t^E, \hat{\lambda}_t^P, \hat{\lambda}_t^E, \hat{\psi}_t^P, \hat{\psi}_t^E, \hat{\mu}_t^P, \hat{\mu}_t^E, \hat{\tilde{R}}_t, \hat{\tilde{N}}_t^P, \hat{\tilde{N}}_t^E, \hat{\tilde{W}}_t^P, \hat{\tilde{W}}_t^E, \\
\tilde{H}_t^P, \tilde{H}_t^I, \tilde{H}_t^E, \tilde{Q}_t^P, \tilde{Q}_t^E, \tilde{r}_t^H, \tilde{r}_t^K, \tilde{I}_t, \tilde{Y}_t, \tilde{B}_t^P, \tilde{B}_t^I, \tilde{B}_t^E, \tilde{A}_t, \tilde{\xi}_t, \tilde{s}_t
\end{bmatrix},
\]

and is characterized by equations (C.1)-(C.30).

### D The solution method

As discussed in the main text, we treat the collateral constraints as inequalities when we solve the model, and add two complementary slackness conditions; (14) and (24), respectively. We then adopt the solution method of Holden and Paetz (2012), on which this appendix builds. In turn, Holden and Paetz (2012) expand on previous work by Laséen and Svensson (2011). With first-order perturbations, this solution method is equivalent to the piecewise linear approach used by Guerrieri and Iacoviello (2014), as discussed by Guerrieri and Iacoviello (2015) and the references therein.\(^{25}\) Finally, Holden and Paetz (2012) and Guerrieri and Iacoviello (2015) evaluate the accuracy of their respective methods against a global solution based on projection methods. This is done for a very simple model with a borrowing constraint, for which a global solution can be obtained. They find that the non-linear local approximations are very accurate. For the model used in this paper, however, the large number of state variables (9 endogenous state variables and 3 shocks) renders the use of global solution methods impractical due to the curse of dimensionality associated with such methods.

The collateral constraints put an upper bound on the borrowing of each of the two constrained agents. While the constraints are binding in the steady state, this may not be the case outside the steady state, where the constraints may be occasionally non-binding. Observe that we can reformulate the collateral constraints in terms of restrictions on each agent’s shadow value of borrowing; \(\mu^j_t, j \in \{I, E\}\). We know that \(\mu^j_t \geq 0\) if and only if the optimal debt level of agent \(j\) is exactly at or above the credit limit. In other words, we need to ensure that \(\mu^j_t \geq 0\). If this restriction is satisfied with inequality, the constraint is binding, so the slackness condition is satisfied. If it holds with equality, the collateral constraint becomes non-binding, but the slackness condition is still satisfied. If instead \(\mu^j_t < 0\), agent \(j\)’s optimal level of debt is lower than the credit limit, so that treating his collateral constraint as an equality implies that we are forcing him to borrow “too much.” In this case, the slackness condition is violated. We then need

\(^{25}\) They also emphasize the equivalence between these two approaches and the widely used extended path algorithm of Fair and Taylor (1983).
to add shadow price shocks so as to “push” \( \mu^j_i \) back up until it exactly equals its lower limit of zero and the slackness condition is satisfied. The idea of adding such shocks to the model derives from Laséen and Svensson (2011), who use such an approach to deal with pre-announced paths for the interest rate setting of a central bank. The contribution of Holden and Paetz (2012) is to develop a numerical method to compute the size of these shocks that are required to obtain the desired level for a given variable in each period, and to make this method applicable to a general class of potentially more complicated problems than the relatively simple experiments conducted by Laséen and Svensson (2011).

We first describe how to compute impulse responses to, say, a technology shock. The first step is to add independent sets of shadow price shocks to each of the two log-linearized collateral constraints. To this end, we need to determine the number of periods \( T \) in which we conjecture that the collateral constraints will be non-binding. This number may be smaller than or equal to the number of periods for which we compute impulse responses; \( T \leq T_{IRF} \). For each period \( t \leq T \), we then add shadow price shocks which hit the economy in period \( t \) but become known at period 0, that is, at the same time the economy is hit by a given shock. In other words, the log-linearized collateral constraints become:

\[
\frac{n^Y}{n^B} \mathbf{B}_t = \mathbf{s} + E_t \{ \mathbf{Q}_{t} \} + \mathbf{H}_t - \beta \mathbf{R}_t - \sum_{s=0}^{T-1} \varepsilon_{s,t-s}^{SP,I}.
\]

\[
\frac{n^Y}{n^B} \mathbf{E}_t \mathbf{E}_t = \mathbf{s} - \beta \mathbf{R}_t + \frac{K}{K + QH} \mathbf{E}_t \{ \mathbf{Q}_{t} \} + \frac{QH}{K + QH} \mathbf{E}_t \{ \mathbf{R}_{t+1} \} + \sum_{s=0}^{T-1} \varepsilon_{s,t-s}^{SP,E},
\]

where \( \varepsilon_{s,t-s}^{SP,j} \) is the shadow price shock that hits agent \( j \) in period \( t = s \), and is anticipated by all agents in period \( t = t - s = 0 \) ensuring consistency with rational expectations. We let all shadow price shocks be of unit magnitude. We then need to compute two sets of weights \( \alpha_{\mu^I} \) and \( \alpha_{\mu^E} \) to control the impact of each shock on \( \mu^I_i \) and \( \mu^E_i \). The “optimal” sets of weights ensure that \( \mu^I_i \) and \( \mu^E_i \) are bounded below at exactly zero. The weights are computed by solving the following quadratic programming problem:

\[
\alpha^* = \begin{bmatrix} \alpha^*_{\mu^I} & \alpha^*_{\mu^E} \end{bmatrix}^t = \arg \min \left[ \begin{bmatrix} \alpha_{\mu^I} & \alpha_{\mu^E} \end{bmatrix} \begin{bmatrix} \mu^I + \tilde{\mu}^I & \alpha_{\mu^I} \end{bmatrix} \begin{bmatrix} \mu^E + \tilde{\mu}^E \end{bmatrix} \right],
\]

subject to

\[
\alpha_{\mu^I} \geq 0,
\]

\[
\mu^I + \tilde{\mu}^I \alpha_{\mu^I} + \tilde{\mu}^j \alpha_{\mu^j} \geq 0,
\]

\( j = \{ I, E \} \). Here, \( \mu^I \) and \( \tilde{\mu}^I \), respectively, the steady-state value and the unrestricted relative impulse response of \( \mu^I \) to a technology shock, that is, the impulse-response of \( \mu^I \) when the collateral constraints are assumed to always bind. In this respect, the vector \( \begin{bmatrix} \mu^I + \tilde{\mu}^I \end{bmatrix} \) contains the absolute, unrestricted impulse responses of the two shadow values stacked. Further, each matrix \( \tilde{\mu}^j \) contains the relative impulse responses of \( \mu^I \) to shadow price shocks to agent \( k \)'s constraint for \( j, k = \{ I, E \} \), in the sense that column \( s \) in \( \tilde{\mu}^j \) represents the response of the shadow value to a shock \( \varepsilon_{s,t-s}^{SP,j} \), i.e. to a shadow price shock that hits in period \( s \) but is anticipated
at time 0, as described above. The off-diagonal elements of the matrix \( \begin{bmatrix} \tilde{\mu}^{I,SP,I} & \tilde{\mu}^{I,SP,E} \\ \tilde{\mu}^{E,SP,I} & \tilde{\mu}^{E,SP,E} \end{bmatrix} \) take into account that the impatient household may be affected if the collateral constraint of the entrepreneur becomes non-binding, and vice versa.

We can explain the nature of the optimization problem as follows. First, note that \( \mu^j + \tilde{\mu}^j A + \tilde{\mu}^{j,SP,j} \alpha_{j,j} + \tilde{\mu}^{j,SP,k} \alpha_{j,k} \) denotes the combined response of \( \mu^j \) to a given shock (here, a technology shock) and a simultaneous announcement of a set of future shadow price shocks for a given set of weights. Given the constraints of the problem, the objective is to find a set of optimal weights so that the impact of the (non-negative) shadow-price shocks is exactly large enough to make sure that the response of \( \mu^j \) is never negative. The minimization ensures that the impact of the shadow price shocks will never be larger than necessary to obtain this. Finally, we only allow for solutions for which the value of the objective function is zero. This ensures that at any given horizon, positive shadow price shocks occur if and only if at least one of the two constrained variables, \( \mu^l \) and \( \mu^E \), are at their lower bound of zero in that period. As pointed out by Holden and Paetz (2012), this can be thought of as a complementary slackness condition on the two inequality constraints of the optimization problem. Once we have solved the minimization problem, it is straightforward to compute the bounded impulse responses of all endogenous variables by simply adding the optimally weighted shadow price shocks to the unconstrained impulse responses of the model in each period.

We rely on the same method to compute dynamic simulations. In this case, however, we need to allow for more than one type of shock. For each period \( t \), we first generate the shocks hitting the economy. We then compute the unrestricted path of the endogenous variables given those shocks and given the simulated values in \( t - 1 \). The paths of the restricted variables then take the place of the impulse responses in the optimization problem. If the unrestricted paths of the bounded variables \( \mu^l \) and \( \mu^E \) never hit the bounds in future periods, our simulation for period \( t \) is fine. If the bounds are hit, we follow the method above and add anticipated shadow price shocks for a sufficient number of future periods. We then compute restricted values for all endogenous variables, and use these as our simulation for period \( t \). Note that, unlike the case for impulse responses, in our dynamic simulations not all anticipated future shadow price shocks will eventually hit the economy, as other shocks may occur before the realization of the expected shadow price shocks and push the restricted variables away from their bounds.

As discussed in the main text, taking occasionally non-binding constraints into account is important also from a quantitative viewpoint, as entrepreneurs become unconstrained as much as 50% of the time for the highest LTV ratios we consider. To shed further light on this aspect, Figure D.1 displays output volatility as a function of the steady-state LTV ratio, as in Figure 2 in the main text, along with the corresponding statistics without accounting for occasionally non-binding constraints, i.e., from a counterfactual model in which the collateral constraints are simply treated as equalities. As the average LTV ratio is raised, the approximation error arising from treating the constraints as always binding increases substantially. In particular, output volatility increases monotonically with the average LTV ratio when credit constraints are treated as always binding. As described in the main text, raising the average LTV ratio involves a trade-off between increasing the exposure of constrained agents to fluctuations in the value of their collateral and decreasing the frequency with which these agents are constrained. An approximation that treats collateral constraints as always binding misses the second leg of this trade-off.

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26 Each matrix \( \tilde{\mu}^{j,SP,k} \) needs to be a square matrix, so if the number of periods in which we guess the constraints may be non-binding is smaller than the number of periods for which we compute impulse responses, \( T < TIRF \), we use only the first \( T \) rows of the matrix, i.e., the upper square matrix.
Figure D.1: *The importance of occasionally non-binding credit constraints.*

Notes. The figure illustrates the effect on output volatility of taking non-binding constraints into account. Numbers are median values from 501 stochastic model simulations of 2000 periods.

### E Baseline parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
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<tr>
<td>Preference parameters</td>
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<td>$\beta^p$</td>
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F Additional figures

Figure F.1: Impulse responses of selected key variables to a positive technology shock.
Figure F.2: Impulse responses of selected key variables to a positive technology shock.
Figure F.3: Impulse responses of selected key variables to a positive land demand shock.
Figure F.4: Impulse responses of selected key variables to a positive land demand shock.
Figure F.5: Impulse responses of selected key variables to a positive credit limit shock.
Figure F.6: Impulse responses of selected key variables to a positive credit limit shock.
Figure F.7: Standard deviations of main variables for different LTV ratios.
Note: See the notes to Figure 2.
Figure F.8: Variance contributions to fluctuations of selected key variables of each of the three shocks in the model for different LTV ratios.

Note: See the notes to Figure 2.