Changing Credit Limits, Changing Business Cycles*

Henrik Jensen
University of Copenhagen and CEPR

Søren Hove Ravn
University of Copenhagen

Emiliano Santoro†
University of Copenhagen

December 2016

Abstract

In the last half century, capital markets across the industrialized world have undergone massive deregulation, involving large increases in the loan-to-value (LTV) ratios of households and firms. We study the business-cycle implications of this phenomenon in an estimated dynamic general equilibrium model with multiple credit-constrained agents. A progressive relaxation of credit constraints initially leads to both higher macroeconomic volatility and stronger comovement between debt and real activity. This pattern reverses at LTV ratios not far from those currently observed in many advanced economies, due to credit constraints becoming non-binding more often. The non-monotonic relationship between credit market conditions and macroeconomic fluctuations carries important lessons for regulatory and macroprudential policymakers. While reducing the average LTV ratio may unintentionally increase macroeconomic volatility, a countercyclical LTV ratio proves to be successful in dampening business cycle fluctuations and, most importantly, avoiding dramatic output drops.

Keywords: Occasionally binding credit constraints, business cycles, capital-market liberalization, macroprudential policy.
JEL: E32, E44.

*We thank Fernando Broner, Gauti Eggertsson, Martin Ellison, Luca Guerrieri, Tom Holden, Dirk Niepelt, Ivan Petrella, Federico Ravenna, Gitte Yding Salmansen, Andrea Tambalotti, and seminar participants at Bank of England, Bank of Finland, Copenhagen Business School, Danmarks Nationalbank, DIW Berlin, University of Copenhagen, University of Lancaster, University of Oxford, the "Workshop on Macroeconomics, Financial Frictions and Asset Prices" at the University of Pavia, the "20th SCE Conference on Computing in Economics and Finance" in Oslo, the "8th Nordic Summer Symposium in Macroeconomics" in Laugarvatn, and the "9th Anglo-French-Italian Workshop" at the Université of Aix-en-Marseille for helpful comments and suggestions. The usual disclaimer applies.

†Corresponding author. Postal address: Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Building 26, 1353 Copenhagen, Denmark. E-mail: Emiliano.Santoro@econ.ku.dk.

1 Introduction

Credit flows are crucial for the functioning of an economy where inhabitants want to alter the profile of purchases over time. Consumers may want to smooth consumption and finance their purchases of durable goods. Likewise, firms may desire to obtain funds for investment projects that only pay off later. Such intertemporal trades are typically plagued by informational problems leading to a multitude of financial market imperfections. One implication is that households and firms become credit constrained, and often have to provide collateral to obtain loans. Under such circumstances, the degree to which credit constraints bind is influential for the economy's response to various disturbances. The main purpose of this paper is to study the business-cycle properties of a conventional DSGE model with credit constraints á la Kiyotaki and Moore (1997) under different credit conditions. Since credit availability is largely determined by how much an agent can borrow against his collateral—the loan-to-value (LTV) ratio—we model changes in credit conditions as changing LTV ratios.

Capital markets have undergone massive deregulation across the industrialized world in the past decades. Looking at the credit market, one dimension of this phenomenon has consisted in a large increase of both household and corporate debt secured by some form of collateral. This has been documented, among others, by Jordá *et al.* (2016) for households and Graham *et al.* (2014) for firms. Figure 1 shows loans relative to assets for households and firms in the U.S. in the post-war period. The observed secular increases are consistent with increased credit availability through higher LTV ratios.[1] To study the effects of this structural transformation on the U.S. business cycle, we incorporate collateral constraints into a real business cycle model with heterogeneous agents, in the vein of Iacoviello (2005), Liu *et al.* (2013), Justiniano *et al.* (2015); *inter alia.* A durable good, land, is used for both consumption purposes and production. In addition, land serves as collateral for "impatient", credit-constrained households, as well as for entrepreneurs. The lenders in the economy are "patient", financially unconstrained households. In contrast to most of the existing business cycle literature, we explore the implications of credit constraints not

[1] As we discuss in Appendix A, the aggregate ratios of loans to assets reported in Figure 1 are likely to understate the actual LTV requirements faced by the marginal borrower. However, while alternative measures may yield higher *levels* of LTV ratios, they give rise to the same conclusions about the development over time of these ratios. See also Taylor (2015), who documents the rise of household and firm borrowing in a sample of 17 advanced economies dating back to 1870.

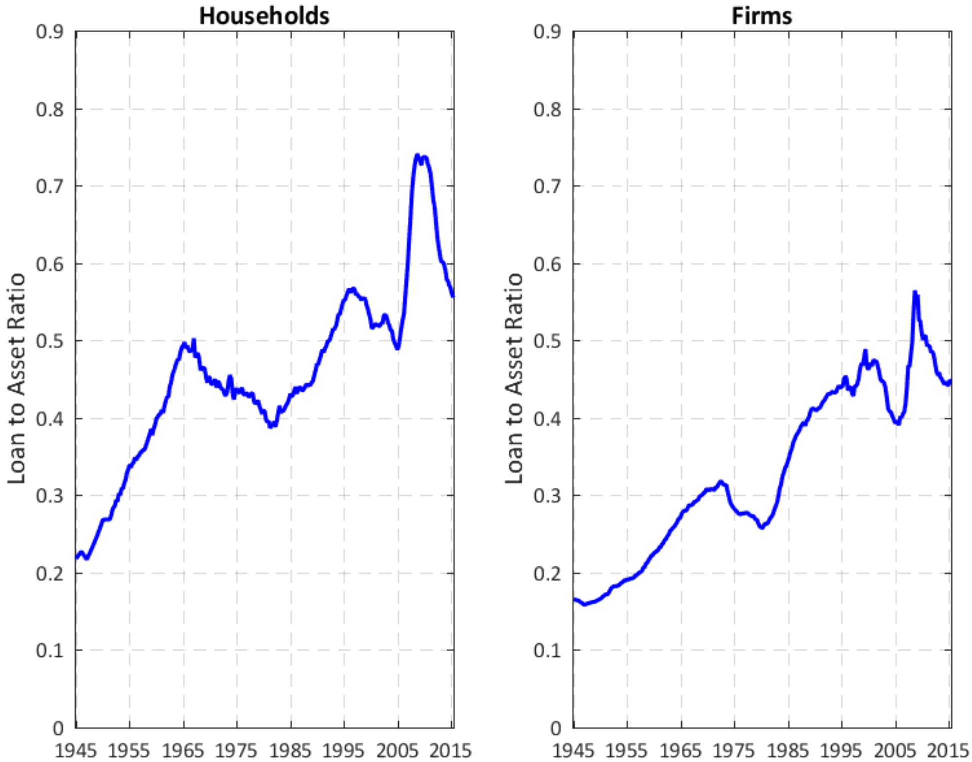

Figure 1: *The ratio of liabilities to assets for households and firms in the United States, 1945Q4–2016Q2.*
Source: See Appendix A.

binding at all points in time.[2] To ensure that the model matches key features of the U.S. economy, we estimate it using the Simulated Method of Moments. This approach also allows us to account for the non-linearities arising from occasionally binding constraints.

Our main findings are that macroeconomic volatility and co-movement between debt and real variables display a hump-shaped pattern in response to changes in the LTV ratio. Starting from relatively low LTV ratios, higher credit limits allow financially constrained agents to succumb to their relatively higher impatience and engage in debt-financed consumption and investment. This reinforces the macroeconomic repercussions of shocks affecting the borrowing capacity of these agents. As a result, output fluctuations become larger and credit issuance becomes more procyclical with a deepening of financial markets. Eventually, a further increase in the LTV ratio reverses this pattern. While credit constraints remain binding after negative shocks to the economy, higher LTV ratios increase the likelihood that credit constraints become non-binding in the face of expansionary shocks. In such cases, the consumption and investment decisions of households and

[2]See Guerrieri and Iacoviello (2016) and Maffezzoli and Monacelli (2015) for two recent contributions which also incorporate occasionally binding credit constraints.

entrepreneurs are delinked from changes in the value of the collateral assets, dampening the volatility of aggregate economic activity and the co-movement between debt and real activity.

The non-monotonic relationship between credit market conditions and macroeconomic fluctuations poses important challenges for regulatory and macroprudential policies. In fact, our analysis establishes a macroeconomic volatility tradeoff that arises from collateral constraints not binding at all points in time. On one hand, a reduction of credit limits may succeed in dampening the asset price sensitivity of those borrowers who remain credit constrained before and after the intervention. This effect is widely acknowledged and represents the main theoretical underpinning of macroprudential policy strategies that have suggested to curb LTV ratios after the 2007–2008 financial crisis (see IMF, 2011 for an extensive survey). On the other hand, lower credit limits increase the frequency at which credit constraints bind, increasing borrowers' sensitivity to fluctuations in credit availability. From a policy perspective, the non-linear relationship between macroeconomic volatility and credit limits questions the adequacy of measures aimed at imposing caps on the LTV ratio. In fact, a macroprudential policymaker might unintentionally raise output volatility by lowering credit limits in a situation where equity requirements are particularly lax.

By contrast, we demonstrate that imposing a countercyclical LTV ratio not only reduces macroeconomic volatility, but is particularly effective at dampening the size of economic contractions, when credit constraints generally bind. Consistent with common definitions of Value-at-Risk, we construct a measure of *GDP-at-risk*, defined as the maximum negative deviation of output from steady state occurring within the top 95 percent of the distribution of output observations (see De Nicolò and Lucchetta, 2013). A countercyclical LTV ratio avoids large output drops and reduces GDP-at-risk by increasing credit availability when it is needed the most: At the turning point of the business cycle, when asset prices and collateral values drop, an automatic relaxation of credit constraints reduces the necessary deleveraging by households and firms.

The analysis of the transmission of different shocks also reveals two other important mechanisms dictated by the structure of our model economy. A notable property is that shocks to land demand only play a minor role for output fluctuations, although they remain the main drivers of land prices. By contrast, technology and financial shocks account for most of output variability. This contrasts with the findings of Liu *et al.* (2013), where entrepreneurial collateralized borrowing greatly amplifies shocks to patient households' land demand. In their setup these shocks emerge as a key driver of investment and, in turn, output fluctuations. In our model, instead, shocks to land demand shift both patient and impatient households' preferences, so that land tends to be 'tied up' away from productive use. This greatly reduces the impact of this shock on output. To that effect, the presence of financially constrained households—which are not featured in Liu *et al.* (2013)—is particularly important, as a positive shock to their preference for land services reinforces their attitude to accumulate land in order to access credit. A second key structural feature of our framework is that technology shocks are amplified by collateral constraints. This is noteworthy, since previous studies have reported that collateral constraints have little or no role in amplifying and propagating shocks to firm productivity, see, e.g., Cordoba and Ripoll (2004) and Liu *et al.* (2013). In our model, a positive technology shock causes an increase in the marginal productivity of land, inducing entrepreneurs to demand more of it. The resulting upward pressure on the land price leads to an increase in the value of both impatient households' and entrepreneurs' collateral assets, causing an endogenous relaxation of their credit constraints. Both agents therefore demand more land, causing further upward movements in expected future credit availability, and so on. This two-sided externality, which is stronger at relatively higher LTV ratios, ensures that more land ends up in the hands of the entrepreneurs, and thus in productive use.

The rest of the paper is organized as follows. The next subsection reviews some relevant literature. Section 2 presents our model, while our solution method, calibration, and estimation are described in Section 3. Section 4 contains the presentation and discussion of our results. Section 5 offers some policy prescriptions to deal with the non-monotonicity of output volatility with respect to the degree of credit market tightness. Section 6 concludes.

1.1 Related literature

Our paper connects to recent contributions that deal with time-variation in the quantity dimension of credit conditions, as measured by the maximum loan-to-value ratio. Some examples are Mendicino (2012a, 2012b) and Walentin (2014). However, they do not consider the possibility of financial constraints being only occasionally binding. In two recent papers, Justiniano *et al.* (2014, 2015) examine the factors behind the expansion of household debt between 2000 and 2007, as well as its subsequent decline. They show that factors affecting house prices directly, along with changes in the credit supply to households, can account for most of the rise in household leverage during the pre-crisis boom, while higher credit limits only played a limited role. However, as they focus on very recent developments in the credit market, they only consider increases in the LTV ratio from a relatively high level (above 0.8). Likewise, Guerrieri and Iacoviello (2016) focus on the recent boom-bust cycle in the U.S. housing market, demonstrating that the macroeconomic sensitivity to house price changes is much smaller during booms—when house prices are high and collateral constraints are therefore likely to be non-binding—than during recessions, when credit constraints typically bind. As a key point of departure from these studies, we do not specifically aim at providing an explanation of the recent macroeconomic and financial turmoil. Instead, we consider the business cycle implications of a secular increase of both household and corporate leverage, as that documented in Figure 1, devising a setup where both households and firms are credit constrained. In fact, we show that corporate debt plays a decisive role in determining non-linearities in both macroeconomic volatility and the cyclicality of private debt. This finding is in line with Maffezzoli and Monacelli (2015), who demonstrate that the macroeconomic effects of financial shocks are highly non-linear in the level of accumulated corporate debt.[3]

Our findings are consistent with some recent empirical studies on the nature of business cycles in a high-leverage environment. Jordà *et al.* (2013) have demonstrated that large build-ups in household and corporate credit during a boom are associated with more severe subsequent recessions. In related work, Schularick and Taylor (2012) have documented that the real effects of financial crises have been significantly larger in modern times as compared to the pre-World War II era, when leverage ratios were low, while Taylor (2015) shows that the strength of the recovery after financially driven recessions—relative to normal recessions—has been much more subdued after World War II, when LTV ratios have been much higher. In line with these empirical results, higher LTV ratios make our model economy more fragile to adverse economic shocks, including financial shocks. Combined with the fact that credit constraints tend to remain binding during downturns, this means

[3]The model of Maffezzoli and Monacelli (2015) does not feature credit-constrained households. For models in which credit constraints apply to both households and firms, see Iacoviello (2005, 2015).

that higher leverage generally leads to deeper recessions. As a direct implication, business cycle asymmetry increases with higher credit limits. In related work we document that, in recent decades, higher credit limits in the U.S. have indeed been accompanied by a more negatively skewed business cycle; cf. Jensen *et al.* (2016). Finally, Jordá *et al.* (2016) have recently documented that the correlation between the growth rate of real credit and that of output and its main components is higher when aggregate leverage is high (i.e., the last several decades) than when it is low (the pre-World War II era); a result broadly in line with our findings concerning business cycle co-movement, although these authors do not study potential non-linearities in this relationship.

The empirical literature has produced no firm consensus on the link between financial deepening and macroeconomic volatility. In support of our findings, Alatrash *et al.* (2014) document an inverse U-shaped relationship between the volatility of output growth and the ratio of private credit to GDP in countries with a high-quality financial sector, with the peak of volatility being reached at credit-to-GDP ratios of around 130 percent. Other studies, however, have reached different conclusions, with Acemoglu *et al.* (2003) and Beck *et al.* (2006) finding no significant effect of financial development on macroeconomic volatility; Denizer *et al.* (2002), Dabla-Norris and Srivisal (2013), and Mallick (2014) reporting that financial deepening tends to dampen business-cycle volatility, and Easterly *et al.* (2000) uncovering a U-shaped relationship between the amount of private credit and output growth volatility. Our contribution is also connected to a large literature suggesting that innovation in the credit market—especially in consumer credit and home mortgages—might have played a role in the so-called Great Moderation; see den Haan and Sterk (2010) for a critical review. According to this view, higher credit limits may have contributed to the decrease in macroeconomic volatility in the period spanning from the mid-1980s until the start of the recent financial crisis. Jordá *et al.* (2016) document that since World War II, and especially since the beginning of the 1970s, the large increase in the ratio of private credit to GDP observed across their sample of advanced economies has coincided with a decline in the volatility of real output and its main components. In our setup, we only observe such a decline beyond relatively high LTV ratios. While this view may thus seem at odds with our findings, it is important to keep in mind that none of the factors typically ascribed to the Great Moderation are actually featured in our model.[4]

[4] These factors include better monetary policy (Boivin and Giannoni, 2006), a drop in the volatility of

2 The model

We devise a real business cycle model with heterogeneous agents and credit limits in the vein of Iacoviello (2005), Liu *et al.* (2013), Justiniano *et al.* (2015); *inter alia*. The economy is populated by entrepreneurs and two types of households. Agents are differentiated by their discount factors. Patient households have the highest discount factor, effectively making them lenders in the economy. Impatient households and entrepreneurs have lower discount factors, and can only borrow up to some proportion of the present value of their assets. Both patient and impatient households work, consume non-durable goods and (durable) land, where the latter can be interpreted as related to housing services. Entrepreneurs only consume non-durable goods, and accumulate both land and physical capital, which they rent to producers. These operate in a perfectly competitive sector, where firms combine labor from both types of households as well as capital and land from entrepreneurs, so as to produce non-durable consumption goods and new capital goods. All types of agents have unit mass.

2.1 Patient and impatient households

The preferences of the households are defined over non-durable consumption, C_t^i, the stock of land, H_t^i, and the fraction of time devoted to labor, N_t^i, where $i \in \{P, I\}$ refers to patient and impatient households, respectively. Household i maximizes

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\beta^i\right)^t \left[\log\left(C_t^i - \rho^i C_{t-1}^i\right) + \varepsilon_t \log H_t^i + \frac{\nu^i}{1-\varphi^i}\left(1 - N_t^i\right)^{1-\varphi^i} \right] \right\}, \quad \varphi^i \neq 1, \tag{1}$$

where ε_t is a preference shock satisfying

$$\log \varepsilon_t = \log \varepsilon + \rho_\varepsilon \left(\log \varepsilon_{t-1} - \log \varepsilon\right) + u_t, \qquad 0 < \rho_\varepsilon < 1, \tag{2}$$

where $\varepsilon > 0$ denotes the steady-state value of ε_t, and where $u_t \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$. Moreover, $0 < \beta^i < 1$ is the discount factor, and $\nu^i > 0$ and $\varphi^i > 0$ denote, respectively, the utility weight and the coefficient of relative risk aversion associated with leisure, whereas $0 \leq \rho^i < 1$ measures the degree of habit formation in consumption. Households' different

economic shocks (Stock and Watson, 2003), and smaller dependence on oil (Nakov and Pescatori, 2010).

impatience is captured by the assumption that $\beta^P > \beta^I$. This ensures that, in the steady state, patient and impatient households act as lenders and borrowers, respectively; cf. Woodford (1986). Utility maximization is subject to the following budget constraint:

$$C_t^i + Q_t \left(H_t^i - H_{t-1}^i \right) + R_{t-1} B_{t-1}^i = B_t^i + W_t^i N_t^i, \tag{3}$$

where B_t^i is the stock of one-period debt held at the end of period t, R_t is the gross real interest rate on debt, Q_t is the price of land in units of consumption goods, and W_t^i is the real wage. Moreover, impatient households are subject to a collateral constraint on borrowing:

$$B_t^I \leq s_t \frac{\mathrm{E}_t \left\{ Q_{t+1} \right\} H_t^I}{R_t}, \tag{4}$$

which states that the maximum borrowable resources equal a fraction s_t of the expected present value of durable goods holdings at the end of period t. This constraint can be rationalized in terms of limited enforcement; cf. Kiyotaki and Moore (1997) and Iacoviello (2005). Lenders are assumed to pay a cost $(1 - s_t)\mathrm{E}_t \left\{ Q_{t+1} \right\} H_t^I$ in period $t + 1$ if they are to repossess the collateral in case of default; hence, they will not lend more than $s_t \mathrm{E}_t \left\{ Q_{t+1} \right\} H_t^I / R_t$ in period t.

The term s_t, the loan-to-value ratio, is assumed to satisfy

$$\log s_t = \log s + \rho_s \left(\log s_{t-1} - \log s \right) + v_t, \qquad 0 < \rho_s < 1, \tag{5}$$

where $v_t \sim \mathcal{N}\left(0, \sigma_s^2\right)$. As our aim is to examine the implications of institutional changes in credit conditions, including regulatory measures, we interpret s, which denotes the steady-state loan-to-value (LTV) ratio, as a proxy for the average stance of credit availability.

Patient households' optimal behavior is described by the standard first-order conditions:

$$\begin{aligned}
\frac{1}{C_t^P - \rho^P C_{t-1}^P} - \frac{\beta \rho^P}{\mathrm{E}_t \left\{ C_{t+1}^P \right\} - \rho^P C_t^P} &= \lambda_t^P, && (6) \\
\nu^P \left(1 - N_t^P \right)^{-\varphi^P} &= \lambda_t^P W_t^P, && (7) \\
\lambda_t^P &= \beta^P R_t \mathrm{E}_t \left\{ \lambda_{t+1}^P \right\}, && (8)
\end{aligned}$$

$$Q_t = \frac{\varepsilon_t}{\lambda_t^P H_t^P} + \beta^P \mathrm{E}_t \left\{ \frac{\lambda_{t+1}^P}{\lambda_t^P} Q_{t+1} \right\}, \tag{9}$$

where λ_t^P is the multiplier associated with (3) for $i = P$. Similarly, optimal behavior of impatient households is described by

$$\frac{1}{C_t^I - \rho^I C_{t-1}^I} - \frac{\beta \rho^I}{\mathrm{E}_t \left\{ C_{t+1}^I \right\} - \rho^I C_t^I} = \lambda_t^I, \tag{10}$$

$$\nu^I \left(1 - N_t^I\right)^{-\varphi^I} = \lambda_t^I W_t^I, \tag{11}$$

$$\lambda_t^I - \mu_t^I = \beta^I R_t \mathrm{E}_t \left\{ \lambda_{t+1}^I \right\}, \tag{12}$$

$$Q_t = \frac{\varepsilon_t}{\lambda_t^I H_t^I} + \beta^I \mathrm{E}_t \left\{ \frac{\lambda_{t+1}^I}{\lambda_t^I} Q_{t+1} \right\} + s_t \frac{\mu_t}{\lambda_t^I} \frac{\mathrm{E}_t \left\{ Q_{t+1} \right\}}{R_t}, \tag{13}$$

where λ_t^I is the multiplier associated with (3) for $i = I$, and μ_t^I is the multiplier associated with (4). Additionally, the complementary slackness condition

$$\mu_t^I \left(B_t^I - s_t \frac{\mathrm{E}_t \left\{ Q_{t+1} \right\} H_t^I}{R_t} \right) = 0, \tag{14}$$

must hold along with $\mu_t^I \geq 0$ and (4).

2.2 Entrepreneurs and firms

The representative entrepreneur has preferences defined over non-durables only (cf. Iacoviello, 2005 or Liu *et al.*, 2013), and maximizes

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\beta^E\right)^t \log \left(C_t^E - \rho^E C_{t-1}^E \right) \right\}, \tag{15}$$

where C_t^E is non-durable consumption, $0 \leq \rho^E < 1$ denotes consumption habits, and where we assume that $\beta^E < \beta^P$. This ensures that the entrepreneurs are borrowers in the steady state. Their budget constraint reads as:

$$C_t^E + I_t + Q_t \left(H_t^E - H_{t-1}^E \right) + R_{t-1} B_{t-1}^E = B_t^E + r_{t-1}^K K_{t-1} + r_{t-1}^H H_{t-1}^E, \tag{16}$$

where I_t denotes investment in physical capital, K_{t-1} is the physical capital stock rented to firms at the end of period $t-1$, and H_{t-1}^E is the stock of land rented to firms. Finally,

r^K_{t-1} and r^H_{t-1} are the rental rates on capital and land, respectively. Capital accumulation is given by the law of motion

$$K_t = (1-\delta) K_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t, \qquad 1 > \delta > 0, \quad \Omega > 0, \tag{17}$$

where we have assumed quadratic investment adjustment costs.

Like impatient households, entrepreneurs are credit constrained, and they borrow using capital and land as collateral:[5]

$$B^E_t \leq s_t \mathrm{E}_t \left\{ \frac{Q^K_{t+1} K_t + Q_{t+1} H^E_t}{R_t} \right\}, \tag{18}$$

where Q^K_t denotes the price of installed capital in consumption units. For simplicity, we assume that households and entrepreneurs are subject to common credit limits.[6]

The optimal behavior of the entrepreneurs is characterized by the first-order conditions

$$\begin{aligned} \frac{1}{C^E_t - \rho^E C^E_{t-1}} - \frac{\beta \rho^E}{\mathrm{E}_t\left\{C^E_{t+1}\right\} - \rho^E C^E_t} &= \lambda^E_t, &(19)\\ \lambda^E_t - \mu^E_t &= \beta^E R_t \mathrm{E}_t\left\{\lambda^E_{t+1}\right\}, &(20) \end{aligned}$$

$$-\lambda^E_t + \psi^E_t \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] = \beta^E \mathrm{E}_t \left\{ \psi^E_{t+1} \Omega \left(\frac{I_{t+1}}{I_t}\right)^2 \left(1 - \frac{I_{t+1}}{I_t}\right) \right\}, \tag{21}$$

$$\begin{aligned} \psi^E_t &= \beta^E r^K_t \mathrm{E}_t\left\{\lambda^E_{t+1}\right\} + \beta^E (1-\delta) \mathrm{E}_t\left\{\psi^E_{t+1}\right\} + \mu^E_t s_t \frac{\mathrm{E}_t\left\{Q^K_{t+1}\right\}}{R_t}, &(22)\\ Q_t &= \beta^E r^H_t \mathrm{E}_t\left\{\frac{\lambda^E_{t+1}}{\lambda^E_t}\right\} + \beta^E \mathrm{E}_t\left\{\frac{\lambda^E_{t+1}}{\lambda^E_t} Q_{t+1}\right\} + s_t \frac{\mu^E_t}{\lambda^E_t} \frac{\mathrm{E}_t\left\{Q_{t+1}\right\}}{R_t}, &(23) \end{aligned}$$

where λ^E_t, μ^E_t and ψ^E_t are the multipliers associated with (16), (18) and (17), respectively.

[5] The importance of real estate as collateral for business loans has recently been emphasized by Chaney *et al.* (2012) and Liu *et al.* (2013).

[6] The ratios of loans to assets in Figure 1 do not suggest large differences between households and firms. In fact, in Jensen *et al.* (2016), we show that it is possible to extract a common trend from the two series. In Iacoviello (2005), the LTV ratio faced by entrepreneurs (0.89) is much higher than that faced by impatient households (0.55), while the opposite is the case in Gerali *et al.* (2010), who set 0.35 for entrepreneurs and 0.7 for households. In sum, in lack of conclusive evidence that LTV ratios faced by firms are systematically higher or lower than those faced by households, we assume that they are equal.

Moreover,

$$\mu_t^E \left(B_t^E - s_t \mathrm{E}_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\} \right) = 0, \tag{24}$$

holds along with $\mu_t^E \geq 0$ and (18). Finally, the definition of Q_t^K implies that

$$Q_t^K = \psi_t^E / \lambda_t^E. \tag{25}$$

Firms operate competitively under a constant-returns-to-scale technology. They rent capital and land from entrepreneurs and hire labor from both types of households, so as to maximize profits. The production technology for output, Y_t, is given by

$$Y_t = A_t \left[\left(N_t^P\right)^\alpha \left(N_t^I\right)^{1-\alpha} \right]^\gamma \left[\left(H_{t-1}^E\right)^\phi K_{t-1}^{1-\phi} \right]^{1-\gamma}, \qquad 0 < \alpha,\ \phi,\ \gamma < 1, \tag{26}$$

with total factor productivity A_t evolving according to

$$\log A_t = \log A + \rho_A \left(\log A_{t-1} - \log A \right) + z_t, \qquad 0 < \rho_A < 1, \tag{27}$$

where $A > 0$ is the steady-state value of A_t, and $z_t \sim \mathcal{N}\left(0, \sigma_A^2\right)$. Optimal factor-demand relations follow from the first-order conditions:

$$\alpha \gamma Y_t / N_t^P = W_t^P, \tag{28}$$

$$(1-\alpha) \gamma Y_t / N_t^I = W_t^I, \tag{29}$$

$$(1-\gamma)(1-\phi) \mathrm{E}_t \left\{ Y_{t+1} \right\} / K_t = r_t^K, \tag{30}$$

$$(1-\gamma) \phi \mathrm{E}_t \left\{ Y_{t+1} \right\} / H_t^E = r_t^H. \tag{31}$$

2.3 Equilibrium

We consider a competitive equilibrium where the markets for labor, capital, loans and land all clear. As its supply is held fixed at H, equilibrium in the the market for land implies that

$$H = H_t^P + H_t^I + H_t^E. \tag{32}$$

Also, the economy-wide net financial position is zero, such that

$$B_t^P + B_t^I + B_t^E = 0. \tag{33}$$

The aggregate resource constraint can be written as

$$Y_t = C_t^P + C_t^I + C_t^E + I_t. \tag{34}$$

An equilibrium is then defined as sequences of quantities and prices, $\{Y_t$, C_t^P, C_t^I, C_t^E, I_t, H_t^P, H_t^I, H_t^E, K_t, N_t^P, N_t^I, B_t^P, B_t^I, $B_t^E\}_{t=0}^{\infty}$ and $\{\lambda_t^P$, λ_t^I, λ_t^E, μ_t^I, μ_t^E, ψ_t^E, r_t^K, r_t^H, Q_t^K, Q_t, W_t^P, W_t^I, $R_t\}_{t=0}^{\infty}$, respectively, which conditional on a sequence of shocks $\{A_t, \varepsilon_t, s_t\}_{t=0}^{\infty}$ and initial conditions, satisfies the optimality conditions [(6), (7), (8), (9), (10), (11), (12), (13), (19), (20), (21), (22), (23), (25), (28), (29), (30), and (31)], the budget and credit constraints [(3) for $i = P, I$, (4), and (18)], as well as the technological constraints and market-clearing conditions [(17), (26), (32), (33), and (34)].

3 Model solution and parameterization

In Appendix B we detail the steady state of the model, while Appendix C describes the log-linearized version, which is solved numerically. Given our assumptions about the discount factors, $\beta^P < \beta^I$ and $\beta^P < \beta^E$, both collateral constraints are binding in the steady state. This follows from (20), which pins down the steady-state real interest rate as $R = 1/\beta^P$. As impatient households and entrepreneurs all have a higher subjective real rate of interest, their consumption levels can only be equalized across time when their collateral constraints bind. However, the optimal level of debt of one or both agents may fall short of the credit limit when the model is not at its steady state (say, in case of a large favorable shock), in which case the collateral constraint will be non-binding. In other words, our model features occasionally binding constraints. To account for this, we explicitly treat the collateral constraints as inequalities, and include the complementary slackness conditions, (14) and (24), in the model.

In practice, we follow the approach of Holden and Paetz (2012), who develop a solution method for log-linearized DSGE models featuring inequalities. Building on Laséen and Svensson (2011), the central idea is to introduce a set of "shadow price shocks", which ensure that each of the complementary slackness conditions is satisfied in each period. If the conditions are violated, the shadow price shocks take on values exactly large enough to make the bounded variables, i.e. the debt levels of credit-constrained agents, equal to their (temporarily) unconstrained value. If the borrowing constraints are already binding, the shadow price shocks are zero. To ensure compatibility with rational expectations, the shocks are added to the model as "news shocks", i.e., they are fully anticipated. We present the details of the solution method in Appendix D.[7]

3.1 Calibration and estimation

We parameterize the model to match the quantitative characteristics of the U.S. business cycle. To this end, we first calibrate a subset of the parameters, and then estimate the remaining parameters using the simulated method of moments (SMM, hereafter). SMM is particularly well-suited for DSGE models involving non-binding constraints or other non-linearities, as these preclude the use of the Kalman filter and thus substantially complicate the application of Bayesian methods.[8] Ruge-Murcia (2012) studies the properties of SMM estimation of non-linear DSGE models, and finds that this method is computationally efficient and delivers accurate parameter estimates. Ruge-Murcia (2007) performs a comparison of SMM with other widely used estimation techniques applied to a basic RBC model, and shows that SMM fares quite well in terms of accuracy and computing efficiency, and is less prone to misspecification issues than Maximum Likelihood-based methods.

3.1.1 Calibrated parameters

We choose to calibrate a subset of the model parameters that can be pinned down using a combination of existing studies and first moments of the data. We interpret one period as a quarter. Therefore, we set $\beta^P = 0.99$, implying an annualized steady-state rate of interest of about 4%. Moreover, we have assumed that impatient households and entrepreneurs have lower discount factors than patient households. In the ballpark of available estimates,

[7] We have verified that our solution method yields identical results to those obtained with the recent approach of Guerrieri and Iacoviello (2015).

[8] See Fernández-Villaverde *et al.* (2016) for a discussion of the particle filter as a potential replacement for the Kalman filter in such cases, or Guerrieri and Iacoviello (2016) for a different filtering scheme based on the extended path algorithm of Fair and Taylor (1983), which facilitates a Bayesian estimation of their model.

we set $\beta^I = \beta^E = 0.97$, implying a rather conservative choice about the relative impatience of borrowers and lenders. We set $s = 0.7$, which implies a steady-state LTV ratio faced by both households and entrepreneurs (Calza *et al.*, 2013 use 0.6, Liu *et al.*, 2013 report 0.75, while Justiniano *et al.*, 2014 set a value of 0.8). The Frisch elasticity of labor supply is given by the inverse of φ^i times the steady-state ratio of leisure to work. Calibrating the latter to 3 for both types of households, a Frisch elasticity of labor supply of 1/3 implies $\varphi^i = 9$, $i = \{P, I\}$. We use $\nu^i = 0.27$ for $i = \{P, I\}$, which implies, as desired, that patient households work 1/4 of their time in steady state, and impatient households slightly more. We set the labor income share of patient households to $\alpha = 0.7$: Iacoviello (2005) obtains an estimate of 0.64 by matching impulse responses from his model with those from a VAR, while Iacoviello and Neri (2010) find a value of 0.79 using Bayesian estimation.

The remaining part of the calibration ensures that the model reproduces a set of "big ratios" of the U.S. economy for the post-1980 period. We set the housing weight in the utility function, ε, so as to match a steady-state ratio of residential land to output of 1.45 (at the annual frequency), as reported by Liu *et al.* (2013). This requires a value of $\varepsilon = 0.0811$. Likewise, we set the parameter ϕ, which multiplied by $(1 - \gamma)$ measures land's share of inputs, to obtain a ratio of commercial land to output of 0.65, reported by the same authors. The implied value of $\phi = 0.1693$ is somewhat higher than estimated by Liu *et al.* (2013). We finally set the parameters γ and δ to match an average capital-output ratio of 1.22 and an average ratio of private nonresidential investment to GDP of 0.13 in the U.S. for the period 1980–2015.[9] This requires a value of $\gamma = 0.7466$, implying a non-labor share in the production function close to 1/4, and a capital depreciation rate of $\delta = 0.0266$. The calibrated parameters are summarized in Panel A of Table 1.

3.1.2 Estimated parameters

We rely on SMM to estimate the remaining model parameters, which include the investment adjustment cost parameter Ω, the parameters measuring habit formation in consumption, ρ^i, and the parameters governing the persistence and volatility of the shocks; $\rho_A, \rho_s, \rho_\varepsilon, \sigma_A, \sigma_s$, and σ_ε. Since we observe only aggregate consumption in the data, we as-

[9] We use the current-cost net stock of fixed private non-residential assets, obtained from Table 1.1 in the Fixed Assets accounts of the Bureau of Economic Analysis. We obtain the corresponding measure for investment from Table 1.5. We use annual GDP at current prices obtained from the Federal Reserve's FRED database.

sume that the habit formation parameter is the same for all agents; $\rho^i = \rho$, $i = \{P, I, E\}$.[10] In the estimation, we use five macroeconomic time series for the U.S. economy spanning the sample period 1980:Q1–2016:Q2: Real GDP, real private consumption, real non-residential investment, real house prices, and the average of the two LTA series reported in Figure 1. We use the HP filter to detrend the data series. We plot the data and provide more details in Appendix E. To estimate the model, the following data moments are used: The standard deviations and first-order autoregressive parameters of each of the five variables, as well as the correlation of consumption, investment, and house prices with output.[11] This gives a total of 13 moment conditions to estimate eight parameters. We match these data moments to their corresponding moments from the model, obtained from a 2000-periods simulation.[12] In the estimation, we impose only very general bounds on parameter values: All parameters are bounded below at zero, and the habit formation parameter along with all AR(1)-coefficients are bounded above at 0.99. We use an identity weighting matrix, since Altonji and Segal (1996) have shown that the use of an optimal weighting matrix leads to biased parameter estimates when method-of-moments estimation is applied to covariances, as is the case here, whereas equally weighted method-of-moments estimation avoids this bias. Finally, standard errors are computed using an application of the delta method. Appendix E contains further details about our estimation strategy.

[10] Unlike the other estimated parameters, ρ also affects the steady state of the model. To account for this, we use the following iterative procedure: We first calibrate the model based on the starting value for ρ used to initiate the estimation ($\rho = 0.7$), cf. Appendix E. Upon estimation, we then recalibrate the model for the estimated value of ρ. As it turns out, this only leads to very small changes in the values of ε and ϕ, while the remaining parameters are virtually unaffected.

[11] We do not include the correlation of the LTA series with output in the estimation. In the data, this correlation is strongly negative, reflecting that when output declines, the denominator of the LTA series drops much faster than the numerator (see, e.g., the spike in the LTA ratios around 2008 observed in Figure 1). In the model, instead, the LTV series is exogenous, and thus not affected by fluctuations in output.

[12] Our simulated sample is thus more than 13 times longer than the actual dataset (which spans 146 quarters). Ruge-Murcia (2012) finds that SMM is quite accurate already when the simulated sample is five or ten times longer than the actual data.

Table 1: Parameter values

Panel A: Calibrated parameters		
Parameter	Description	Value
β^P	Discount factor, patient households	0.99
β^i, $i = \{I, E\}$	Discount factor, impatient agents	0.97
φ^i, $i = \{P, I\}$	Curvature of utility of leisure	9
ν^i, $i = \{P, I\}$	Weight of labor disutility	0.27
ε	Weight of housing utility	0.0811
ϕ	Non-labor input share of land	0.1693
γ	Labor share of production	0.7466
δ	Capital depreciation rate	0.0266
α	Income share of patient households	0.7

Panel B: Estimated parameters		
Parameter	Description	Value
Ω	Investment adjustment cost parameter	2.7025 (1.3103)
ρ	Habit formation in consumption	0.2121 (0.0410)
ρ_A	Persistence of technology shock	0.9781 (0.0823)
ρ_s	Persistence of credit-limit shock	0.9781 (0.0107)
ρ_ε	Persistence of land-demand shock	0.99 (0.0110)
σ_A	Std. dev. of technology shock	0.0075 (0.0006)
σ_s	Std. dev. of credit-limit shock	0.0187 (0.0004)
σ_ε	Std. dev. of land-demand shock	0.0444 (0.0327)

Note: For estimated parameters, standard errors are reported in brackets.

The estimated parameters are reported in the Panel B of Table 1. These are generally in line with the existing literature on estimated DSGE models. The estimated value of Ω is well within the range of empirical estimates, which range from nearly to zero in Liu *et al.* (2013) to above 10 in Christiano *et al.* (2014). The parameter measuring habit formation, ρ, is somewhat lower than the available estimates, which are typically in the range between 0.5 and 0.7; see, e.g., Liu *et al.* (2013) or Guerrieri and Iacoviello (2016). The volatility and persistence of the technology shock is close to the estimates of Jermann and Quadrini (2012), and are also in line with values applied in the real business cycle literature (see., e.g., Mandelman *et al.*, 2011). The persistence of the credit limit shock is close to the estimates of Jermann and Quadrini (2012) and Liu *et al.* (2013), while we find this shock to be somewhat more volatile than that reported by these authors. The standard deviation and persistence of the land-demand shock are both quite close to the values reported by Iacoviello and Neri (2010) and Liu *et al.* (2013). Note that the persistence of the land-demand shock is the only parameter to reach any of the bounds imposed in the estimation. The implied business cycle moments and their empirical counterparts are shown in Table 2 in Appendix E. In general, the estimated model succeeds in delivering a close match with the empirical moments, although some variables display less persistence than in the data.

4 Business cycles and credit limits

We now focus on the behavior of key business cycle statistics under different credit limits. We first devise a 'comparative statics' exercise to understand the effects of a relaxation in the degree of credit market tightness on the propagation of shocks to the economy. To this end, we let s vary over the range [0.5,0.9], while leaving other parameters at their calibrated/estimated value.[13]

According to Figure 2, output volatility increases in the average LTV ratio up to $s \approx 0.8$, a value in line with the credit conditions on mortgage loans in a number of advanced countries (see, e.g., Calza *et al.*, 2013). Moving right from the lower bound of the support for s, an increase in the average LTV ratio implies that, *ceteris paribus*, credit-constrained agents acquire higher borrowing capacity, so as to satisfy their relative impatience. As a result, financially-constrained households and entrepreneurs experience a proportionally larger expansion in their leverage in reaction to shocks that inflate their collateral values, as well as a more dramatic deleveraging in the face of contractionary shocks. This inevitably translates into larger fluctuations in debt-financed consumption and investment choices, as compared with economies featuring relatively lower credit limits.

[13] We discretize this interval for 9 different values of s. We set the upper bound of this range so that given the standard deviation of the process for s_t, the actual LTV ratio is bounded from above by 1 with 90 pct. probability.

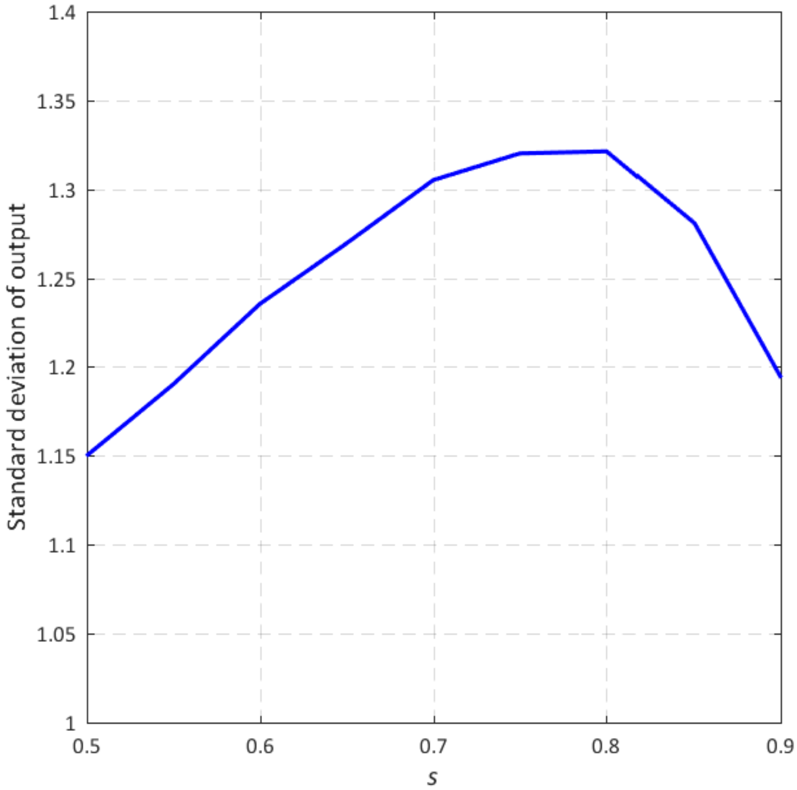

Figure 2: *Standard deviation of output for different LTV ratios.*
Note: Numbers are median values from 501 stochastic model simulations of 2000 periods.

Beyond $s \approx 0.8$ output volatility declines in the LTV ratio. This drop results from the increasing frequency of episodes in which borrowers meet slack financial constraints. To support this claim, Figure 3 shows that borrowers find themselves financially unconstrained more and more often beyond a certain agent-specific threshold (about 0.5 for the entrepreneurs and 0.8 for impatient households). In fact, entrepreneurs (impatient households) become unconstrained as much as two thirds (one fifth) of the time at the upper end of the support for s. Laxer credit conditons imply that, *ceteris paribus*, ex-ante constrained agents face a higher probability of behaving—at least temporarily—as standard consumption smoothers. This translates into dampened fluctuations of both consumption and investment choices.[14] As a result, we observe lower output volatility at relatively high values of s, as well as weaker co-movement between credit and economic activity, as we will discuss in Subsection 4.2.[15]

[14]Figure F.1 in Appendix F shows that an inverse U-shaped pattern emerges also for the volatility of investment and aggregate consumption.

[15]By contrast, Mendicino (2012a) finds that the amplification of technology shocks through collateral constraints increases in the credit limit, but tends to vanish as the LTV ratio approaches one. This happens as fully efficient debt enforcement is possible in her simple Kiyotaki and Moore (1997) economy.

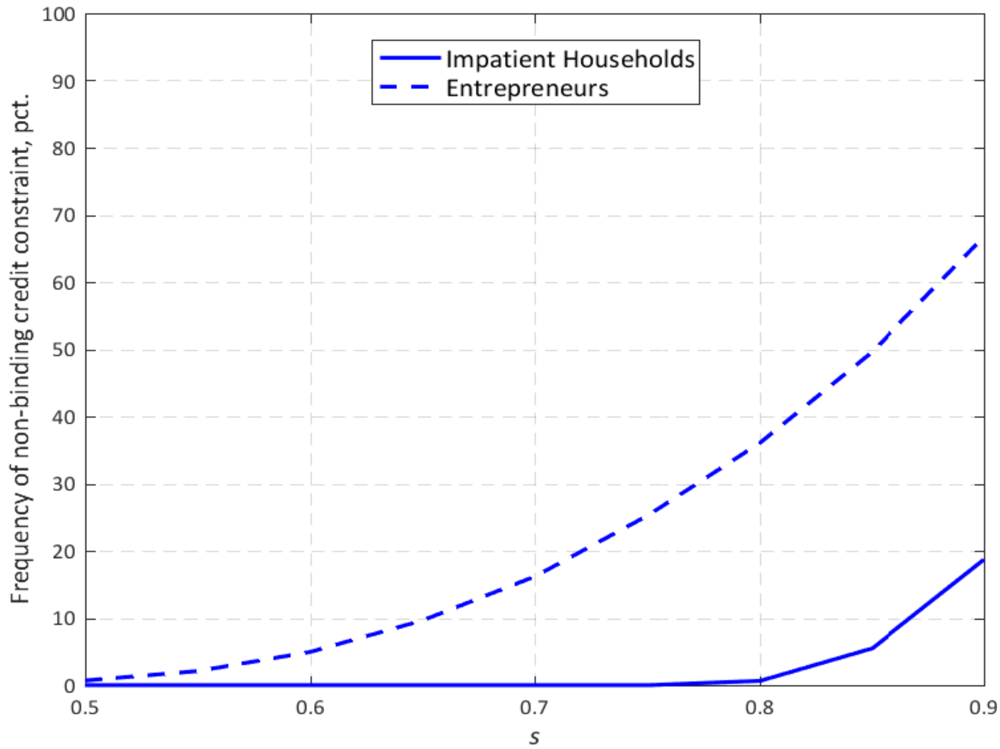

Figure 3: *Frequency of episodes of non-binding constraints for each agent.*
Note: See the notes to Figure 2.

4.1 Explaining non-linearities

To examine the non-linear behavior of macroeconomic volatility in further detail, we look at the propagation of different shocks. We first report a set of numerical results that account for the increase in volatility observed in Figure 2. To this end, we contrast the responses of output to different shocks under alternative degrees of credit market tightness. We also report analogous responses under an alternative model, where credit-constrained agents may only borrow up to their steady-state level of debt. Next, we examine the reversal in output volatility. To this end, we study the propagation of 'large' shocks, which bear a higher potential of making the borrowing constraints non-binding.

4.1.1 Impulse-response analysis

It is worth noting that endogenous credit limits tend to amplify technology shocks, as compared with an alternative economy where endogenous collateral effects are shut off, and more so as the average LTV ratio increases.[16] This result, portrayed by Figure 4,

[16] See also Figures F.2-F.4 in Appendix F, which show the responses of six key variables to a technology shock, a land demand shock, and a credit limit shock, respectively.

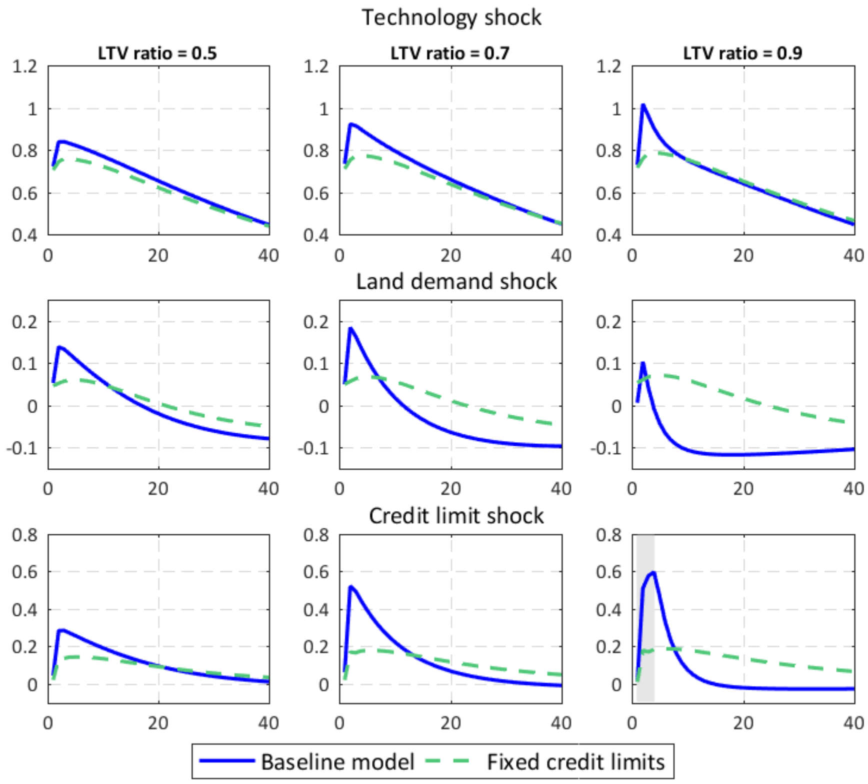

Figure 4: *Impulse responses of output (in percentage deviation from steady state) to a one standard deviation positive technology shock (row 1), a positive land-demand shock (row 2), and a positive credit-limit shock (row 3) for three different LTV ratios;* $s = 0.70$ *(left column),* $s = 0.80$ (center), *and* $s = 0.90$ *(right column).*

Notes: Solid lines denote the baseline model, dashed lines denote the model with fixed credit limits. Light-grey periods are ones in which the entrepreneurs become unconstrained. Impatient households always remain constrained in these experiments.

contrasts with recent business cycle literature that reports little or no amplification of shocks to technology through credit constraints; cf. Kocherlakota (2000), Cordoba and Ripoll (2004), *inter alia*. Also Liu *et al.* (2013) report the effect of different shocks in a model with a fixed credit limit. In their case the response to the technology shock overlaps with its counterpart under endogenous credit limits, while only land-demand shocks display sizable amplification. Such a discrepancy crucially rests on the presence of two types of borrowers in our economy, as compared with Liu *et al.* (2013), where only one borrower (i.e., entrepreneurs) is envisaged. After a positive technology shock, the increase in the marginal productivity of land induces entrepreneurs to demand more of it. The resulting upward pressure on the land price leads to an increase in the value of *both* impatient households' and entrepreneurs' collateral assets, causing an endogenous relaxation of their credit constraints. Therefore, both agents demand more land, causing further upward pressure in future credit availability, and so on. This externality, which is stronger at relatively higher LTV ratios, ensures that more land ends up in the hands of the entrepreneurs, and thus in productive use.

Figure 4 reveals a striking feature of the transmission of land-demand shocks. Compared with other sources of exogenous perturbation, these shocks have a rather short-lived impact on aggregate activity, eventually becoming contractionary despite the very high persistence of the shock itself. This pattern is particularly evident at very high LTV ratios. In this respect, the transmission mechanism in our model contrasts with that of Liu *et al.* (2013), who find that in the presence of competing demand for land by patient households and entrepreneurs, the collateral constraint faced by the latter provides a powerful amplification of land demand. This is not the case in our setup, due to the presence of an additional type of constrained agent—impatient households—who, along with accumulating land in accordance with their preferences, also employ it as collateral in their borrowing activity. As a consequence, after a land-demand shock impatient households increase their stock of land in a very persistent manner. This implies that land is eventually diverted away from firms, and thus from productive use, explaining the drop in output.[17]

Finally, it is worth noting that shocks to credit limits induce an even greater amplification, due to their direct impact on constrained agents' borrowing capacity. Note that for an average LTV ratio of $s = 0.9$, this type of shock makes the entrepreneurs temporarily unconstrained for 3 periods. Section 4.1.2 discusses this specific point, comparing the transmission of contractionary and expansionary shocks of the same absolute size.

An alternative way to assess the role of each specific shock for aggregate dynamics is to compute their relative contribution to output fluctuations. To this end, it is possible to devise a simple measure of our three structural shocks' relative contribution to the volatility of various aggregates. For a generic variable x, we define the contribution of

[17] Another result emerging from the second row of Figure 4 is that the impact effect of a land-demand shock is larger at $s = 0.7$ than at $s = 0.9$. The reason is that, at very high LTV ratios, the wealth effect induced by an increase in the collateral value is such that impatient households' labor supply contracts, thus driving up the marginal product of labor and reducing investment. While this effect in isolation tends to dampen business cycle volatility at very high LTV ratios even in the absence of occasionally non-binding credit constraints, it plays no role for our main results, since land-demand shocks account for virtually none of the variation in output at LTV ratios around our baseline calibration of $s = 0.7$, as we discuss below.

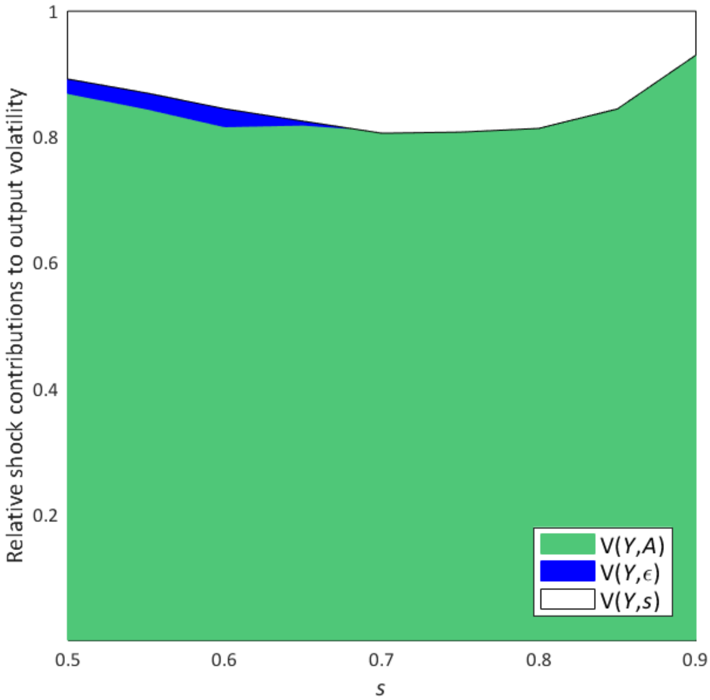

Figure 5: *Variance contribution to output fluctuations of each of the three shocks in the model for different LTV ratios.*
Note: See the notes to Figure 2.

shock ξ to its variance as

$$\mathrm{V}(x,\xi) \equiv \frac{\mathrm{var}[x] - \mathrm{var}[x]_{-\xi}}{3\mathrm{var}[x] - \sum_{\xi} \mathrm{var}[x]_{-\xi}}, \qquad \xi = A, \varepsilon, s, \tag{35}$$

where $\mathrm{var}[x]_{-\xi}$ is the unconditional variance of x, when the structural shock ξ is turned off.[18]

Technology and financial shocks respectively account for about 81% and 19% of output volatility, at our baseline calibration of $s = 0.7$. Figure 5 shows that technology shocks emerge as the main driver of real activity, while financial shocks play a significant role mainly at levels of the average LTV ratio close to the calibrated value.[19] In fact, their importance increases over the range of LTV ratios up to $s \approx 0.7$. Beyond this point, the contribution of financial shocks reduces, consistent with credit constraints becoming

[18] We use this measure since our numerical solution method does not allow us to carry out a standard variance decomposition. However, note that for a linear model, the $\mathrm{V}(x,\xi)$s are equal to the ratios found by conventional variance decompositions.

[19] As expected on a priori grounds, these shocks are the predominant driver of debt fluctuations, no matter the average LTV value, cf. Figure F.5.

non-binding more frequently. Strikingly, land-demand shocks contribute very little to the volatility of output at relatively low values of the average LTV ratio, while playing no role at higher credit limits.[20] This property is intimately linked to the role of impatient households in our framework. As discussed above, these agents effectively tie up land away from productive use after a positive land-demand shock, due to their attitude to durables accumulation both for utility and collateralization motives. By contrast, in the model of Liu *et al.* (2013), where impatient households are not featured, land-demand shocks are shown to account for a substantial fraction of output fluctuations.

4.1.2 Responses to 'large' shocks

In this subsection we temporarily depart from our estimation, perturbating the model with large shocks of each type. Figure 6 displays the response of output to a set of 3-standard deviations, positive shocks, as well as the mirror image of the response to equally-sized negative shocks, under alternative values of the average LTV ratio. This exercise is simply aimed at building intuition on the functioning of occasionally binding constraints, conditional on each separate shock. In our stochastic simulations, instead, combinations of positive shocks in line with our estimation are sufficient to make the constraints non-binding.

At low LTV ratios, collateral constraints remain binding at all times. Therefore, the model economy is still linear, with positive and negative shocks producing the same absolute effect on output. However, as the average LTV ratio increases, this generally raises the likelihood of financial constraints becoming slack in the face of expansionary shocks, as indicated by the grey areas in Figure 6. Notably, positive financial shocks bear a high potential of making the constraint non-binding—and more so for entrepreneurs—due to their direct impact on the collateral value. However, at $s = 0.9$ all types of shocks lead to episodes of non-binding constraints. This is in line with the implications of Figure 3. As discussed above, episodes of non-binding credit constraints dampen the magnitude of the resulting boom, as impatient households and entrepreneurs choose not to exhaust their increased borrowing capacity. The resulting attenuation of economic expansions is the key driving force behind the drop in overall volatility at high average LTV ratios documented

[20] However, as Figure F.5 in Appendix F shows, land-demand shocks represent the main source of variation in the land price.

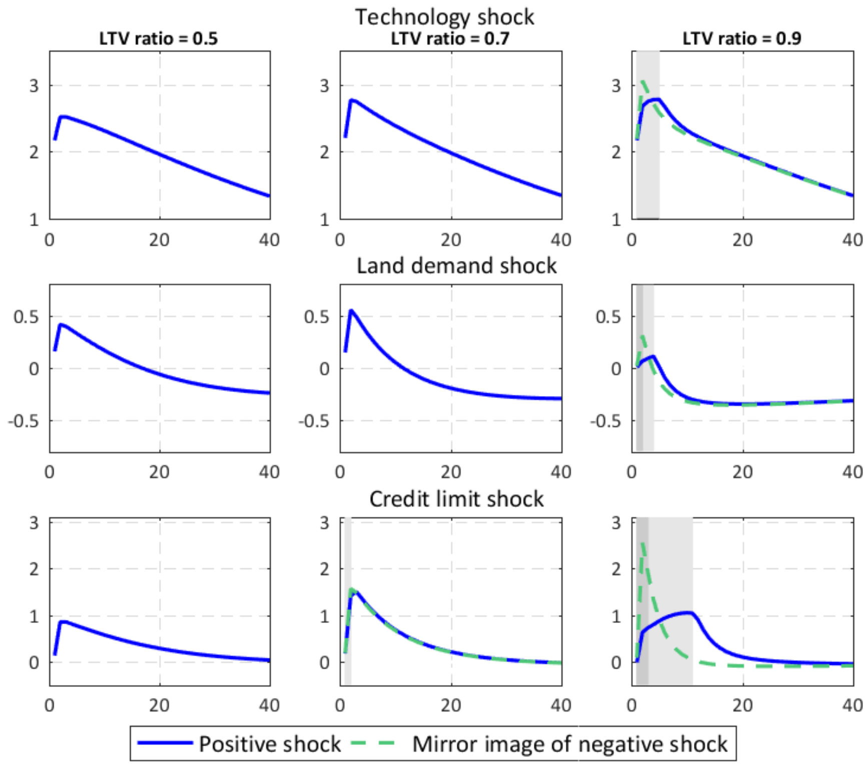

Figure 6: *Impulse responses of output to large (3 standard deviations) shocks to technology (row 1), land demand (row 2), and credit limits (row 3) for three different LTV ratios;* $s = 0.70$ *(left column),* $s = 0.80$ (center), *and* $s = 0.90$ *(right column).*

Notes: Light-grey periods are ones where the entrepreneurs are unconstrained; dark-grey periods are ones where both agents are unconstrained. Solid lines are the impulse response to a positive shock, dashed lines are the mirror image (i.e., the negative) of a similar-sized negative shock.

in Figure 2. On the other hand, in the face of contractionary shocks both types of borrowers remain financially constrained at all levels of the average LTV ratio, making their debt reduction increasingly burdensome.[21] As a result, economic contractions generally become deeper at high average LTV ratios. An implication of this finding is that the business cycle becomes negatively skewed. In related work (Jensen *et al.*, 2016), we present evidence that the U.S. business cycle has in fact become more negatively skewed over the last decades, suggesting that looser credit conditions may indeed have been a driver of this tendency.

[21] In our stochastic simulations, however, at high LTV ratios episodes of non-binding constraints may occur also during contractions, contributing further to the reduction in macroeconomic volatility. Such situations may arise if, e.g., a negative technology shock coincides with a positive credit limit shock that makes the borrowing constraint slack.

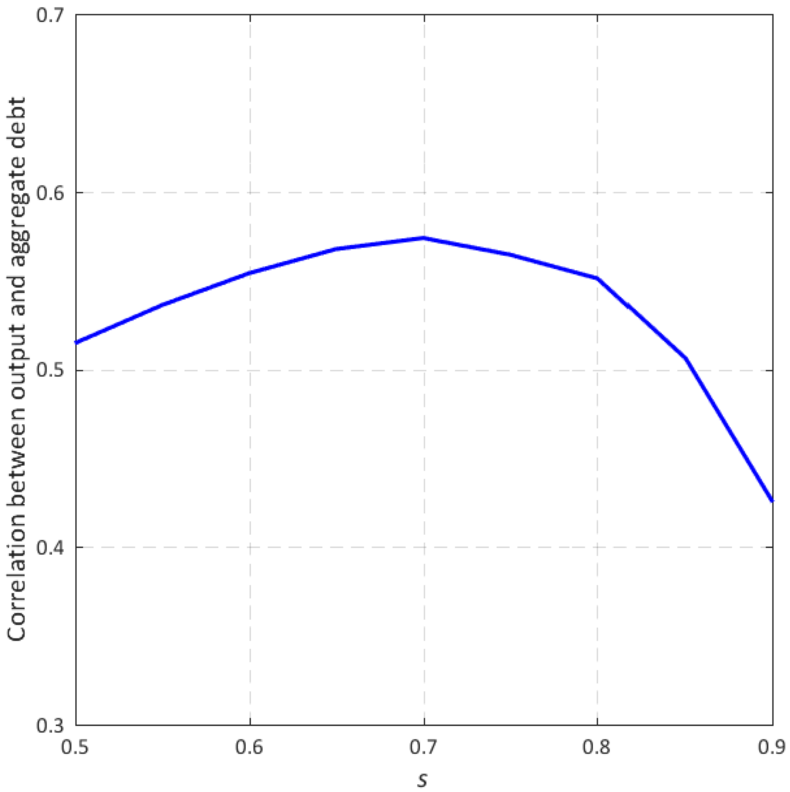

Figure 7: *Correlation coefficient between output and aggregate debt for different LTV ratios.*
Notes: See the notes to Figure 2.

4.2 Co-movement between debt and output

Both the theoretical and the empirical literature have emphasized the role of co-movement between real activity and credit to understand the effect of financial liberalization on the business cycle (den Haan and Sterk, 2010; Campbell and Hercowitz, 2011). This subsection examines the connection between credit limits and the degree of procyclicality of private debt. It turns out that the enhanced ability of impatient households and entrepreneurs to engage in debt-financed consumption and investment has important implications for the co-movement between credit extension and real economic activity.

Figure 7 plots the correlation between aggregate debt and output: This increases in the average LTV ratio up to $s \approx 0.7$, thus reverting as the average LTV ratio approaches its upper bound and credit constraints become non-binding more frequently. To understand this pattern, it is useful to study how each type of agent makes consumption and investment decisions under different credit limits. Focusing on entrepreneurs, Figure 8 shows that the correlation between consumption and debt of entrepreneurs is high when the average LTV ratio is relatively low, but declines at higher LTV ratios. At relatively low values of the

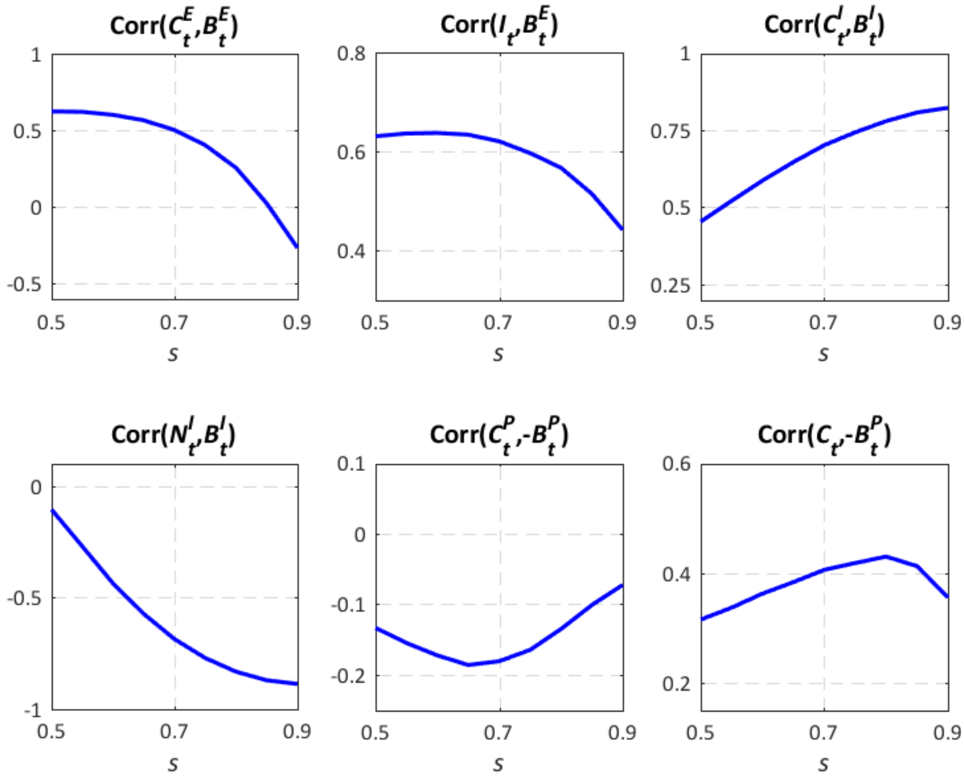

Figure 8: *Correlation coefficients between key variables for different LTV ratios.*
Notes: See the notes to Figure 2.

average LTV ratio, a marginal relaxation of the collateral constraint allows this type of agent to increase its consumption by accessing more credit. However, as the average LTV ratio increases, entrepreneurs find themselves unconstrained more and more often. This reduces the correlation between entrepreneurial consumption and debt, as occasionally binding constraints produces a delinking between debt and consumption dynamics. A similar reasoning applies to the co-movement between debt and investment in capital goods, although the correlation between these variables displays less sizeable changes.

Like the entrepreneurs, also impatient households exploit higher credit limits to increase debt-financed consumption. However, we have seen that impatient households find themselves financially unconstrained less often, even at the upper end of the support of the average LTV ratio. As a result, the correlation between consumption and debt of impatient households increases monotonically, albeit more slowly as s approaches its upper limit; cf. Figure 8. It is worth noting that impatient households behave in accordance with the "financial labor supply accelerator" channel of Campbell and Hercowitz (2009, 2011), according to which looser credit conditions weaken the co-movement between B_t^I and N_t^I;

cf. Figure 8: As credit limits become more lax, households are no longer forced to work longer hours in order to increase their spending in response to a generic positive shock, but may instead rely on access to credit. In our model with multiple agents and input factors, however, this effect does not translate into a weakened co-movement between aggregate debt and output.[22]

The intertemporal choices of patient households mirror those of their credit-constrained counterparts in the debt market. Indeed, starting from low average LTV ratios, the correlation between consumption and savings (i.e., aggregate credit) declines up to $s \approx 0.65$. This indicates that patient households are increasingly willing to postpone consumption, while lending their available resources. Beyond this point, as down-payment requirements drop further and credit constraints are relaxed, borrowers' demand for additional credit slows down, and so does patient households' propensity to save. This reverses the correlation pattern. Finally, Figure 8 shows that the co-movement between aggregate consumption (denoted C_t) and aggregate debt displays a pattern roughly similar to that between output and debt. To sum up, greater credit availability initially allows financially-constrained agents to give vent to their impatience, resulting in a marked increase in the co-movement of real activity and debt, which is only reverted when the economy is flooded with credit availability and instances of non-binding collateral constraints become more frequent.

5 Macroprudential policy implications

In the face of the 2007–2008 financial crisis, several authorities have reacted by introducing limits to the LTV ratios for mortgages (IMF, 2011). Cerutti *et al.* (2015) survey the use of macroprudential policy tools in 119 countries, and report that around one fifth of these have enforced restrictions on LTV ratios. Among other countries, Hong Kong, the Netherlands, New Zealand, Singapore, Sweden have imposed limits to the LTV ratio as the main macroprudential tool; cf. Darbar and Wu (2015).[23] However, the non-linear relationship between macroeconomic volatility and credit limits highlighted in Figure 2

[22] This contrasts with the framework of Campbell and Hercowitz (2009, 2011), where labor of credit-constrained households equals output.

[23] Jácome and Mitra (2015) examine the implementation of LTV limits (along with limits to the debt-service-to-income ratio) in six countries, showing that they were effective in reducing loan-growth and improving debt-servicing performances of borrowers, while experiencing more difficulties in curbing house price growth.

questions the adequacy of such measures, as it implies that macroprudential policymakers might unintentionally raise output volatility by lowering the LTV ratio in situations characterized by lax equity requirements. On one hand, a reduction of credit limits may succeed in dampening the asset price sensitivity of borrowers' spending and investment decisions when they are credit constrained before and after the intervention. On the other hand, lower credit limits increase the frequency at which credit constraints bind, subjecting borrowers more heavily to fluctuations in credit availability. At very high LTV ratios, we have seen that the latter effect dominates.

As an alternative to imposing a cap on the LTV ratio, several academics and policymakers have suggested to reduce the amplitude of housing and financial boom-bust cycles through the introduction of a countercyclical financial sector regulation. In fact, both the Basel Committee on the Global Financial System (2010) and the IMF (2011) have suggested that the LTV ratio could be envisaged as an automatic stabilizer to be adjusted countercyclically around a certain cap, while Lambertini *et al.* (2013) have shown that such a policy can be effective in curbing fluctuations in output as well as household debt. This view is supported by widespread evidence showing that the institutional characteristics of mortgage finance strongly contribute to the procyclicality of the housing market (see, e.g., Calza *et al.*, 2013).

We now examine the stabilizing properties of this type of policy in our model. To this end, we expand (5), so as to account for a systematic response of the LTV ratio to log-deviations of gross output from its steady state:

$$\log s_t = \log s - \omega \log \left(\frac{Y_t}{Y} \right) + \rho_s \left(\log s_{t-1} - \log s \right) + v_t, \quad \omega > 0, \tag{36}$$

where ω indexes the degree of countercyclicality. The left-hand panel of Figure 9 reports the results of our policy exercise for different values of the steady-state LTV ratio and alternative degrees of countercyclicality. As expected, output volatility decreases in ω, at all values of the average LTV ratio. Moreover, the non-monotonic relationship between output volatility and the LTV ratio vanishes at high values of ω, implying that the rule is effective at defusing the effects of occasionally binding constraints. This is confirmed by the right-hand panel of Figure 9, which focuses on the relative volatility of expansionary and contractionary episodes by reporting the interquartile range of the distributions of each of

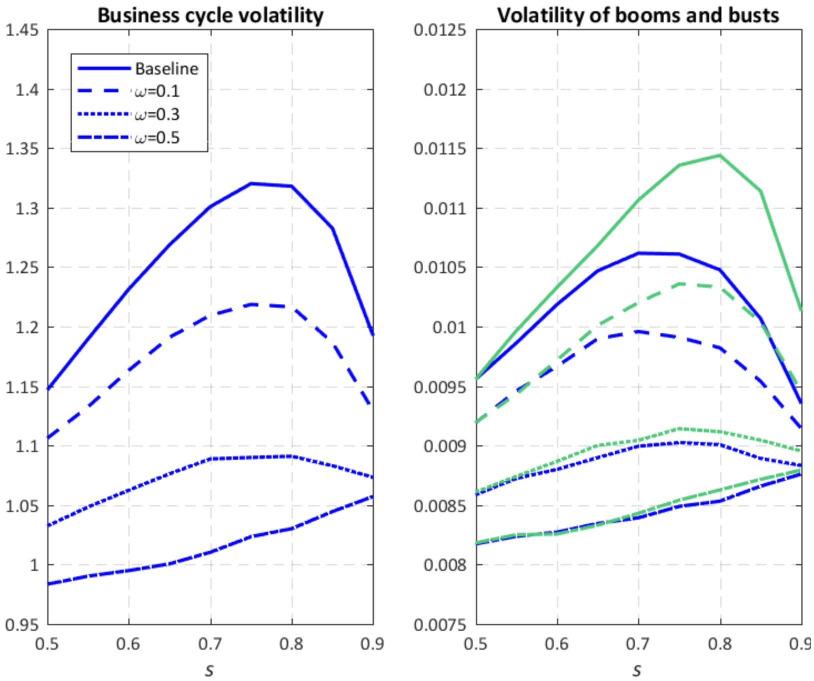

Figure 9: *Left panel: Standard deviation of output for different degrees of countercyclical LTV ratios. Right panel: Interquartile range for output contractions (green lines) and expansions (blue lines) for different degrees of countercyclical LTV ratios.*

Notes: In the right panel we split our simulated samples into expansions and contractions based on whether output is above or below its steady-state level, after which we compute the interquartile range for each of the two subsamples. See also the notes to Figure 2.

these: In the baseline scenario the volatility of expansions is considerably lower than that of contractions, and declines over a wide range of average credit limits, whereas the volatility of contractions declines only at the very end of the support for s. Notably, implementing (36) is effective at reducing the amplitude of contractions by more than that of expansions, counteracting the macroeconomic asymmetry caused by occasionally binding constraints. In fact, for a sufficiently countercyclical rule, contractions and expansions become almost symmetric.

Arguably the key objective of macroprudential policy is to minimize the likelihood of large output drops, rather than smoothing output fluctuations in either direction. For this reason, we now focus our policy analysis on a measure of *GDP-at-risk*, as in De Nicolò and Lucchetta (2013). Consistent with common definitions of Value-at-Risk, we define GDP-at-risk as the maximum negative deviation of output from steady state occurring within the top 95 percent of the distribution of output observations. We therefore look at the 5th percentile of the distribution of output in our stochastic simulations, which—rescaled by the steady-state level of output—is displayed in Figure 10 for a range of steady-state LTV

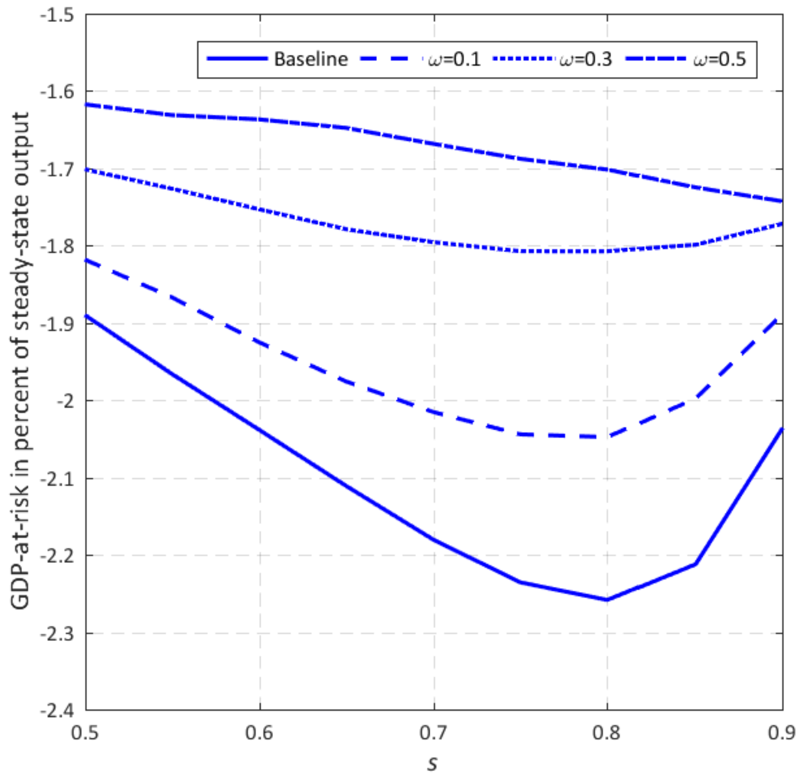

Figure 10: *GDP-at-risk for different degrees of countercyclical LTV ratios.*
Note: See the notes to Figure 2.

ratios and degrees of countercyclicality. At our baseline value of $s = 0.7$, GDP-at-risk in our baseline model is 2.18 percent of steady-state output. The implementation of a countercyclical LTV ratio leads to a quantitatively important attenuation of GDP-at-risk at all steady-state LTV ratios, while alleviating its hump-shaped pattern observed under a constant LTV ratio. In fact, depending on the degree of countercyclicality, this policy reduces GDP-at-risk by between 7 and 24 percent at $s = 0.7$. In other words, the state-contingent rule (36) avoids large output drops by increasing credit availability when it is needed the most: As the onset of an economic contraction leads to a decline in asset prices and collateral values, an automatic relaxation of credit constraints reduces the necessary deleveraging by households and firms.

6 Concluding remarks

We construct a DSGE model with heterogeneous agents and multiple credit constraints, and show that looser credit conditions initially generate increasing business cycle volatility and stronger co-movement between private debt and real economic activity. The pattern reverses at LTV ratios not far from those currently observed in many advanced economies. These non-monotonic relationships are intertwined with the possibility of credit constraints becoming non-binding the higher are credit limits, which in itself paves the way for asymmetric business cycles. While this pattern implies that a simple cap on the average LTV ratio may increase macroeconomic volatility, we show that a countercyclical LTV ratio dampens volatility and reduces the risk of large output drops.

References

Acemoglu, D., S. Johnson, J. Robinson, and Y. Thaicharoen, 2003, Institutional Causes, Macroeconomic Symptoms: Volatility, Crises and Growth, Journal of Monetary Economics 50, 49–123.

Alatrash, Y., D. Leff, T. Minten, M. Soupre, and D. Van Schoot, 2014, Financial Development and Macroeconomic Volatility, mimeo, Barcelona Graduate School of Economics and Universitat Pompeu Fabra.

Altonji, J. G., and L. M. Segal, 1996, Small-Sample Bias in GMM Estimation of Covariance Structures, Journal of Business and Economic Statistics 14, 353–366.

Basel Committee on the Global Financial System, 2010, Macroprudential Instruments and Frameworks: a Stocktaking of Issues and Experiences, CGFS Papers 38, Bank for International Settlements, Basel.

Bates, T. W., K. M. Kahle, and R. M. Stulz, 2009, Why Do U.S. Firms Hold So Much More Cash than They Used To?, Journal of Finance 64, 1985–2021.

Beck, T., H. Degryse, and C. Kneer, 2014, Is More Finance Better? Disentangling Intermediation and Size Effects of Financial Systems, Journal of Financial Stability 10, 50–64.

Boivin, J., and M. Giannoni, 2006, Has Monetary Policy Become More Effective?, Review of Economics and Statistics 88, 445–462.

Boz, E., and E. Mendoza, 2014, Financial Innovation, the Discovery of Risk, and the U.S. Credit Crisis, Journal of Monetary Economics 62, 1–22.

Calza, A., T. Monacelli, and L. Stracca, 2013, Housing Finance and Monetary Policy, Journal of the European Economic Association 11, 101–122.

Campbell, J. R. and Z. Hercowitz, 2009, Welfare Implications of the Transition to High Household Debt, Journal of Monetary Economics 56, 1–16.

Campbell, J. R. and Z. Hercowitz, 2011, The Financial Labor Supply Accelerator, Federal Reserve Bank of Chicago, Working Paper 2011–5.

Cerutti, E., S. Claessens, and L. Laeven, 2015, The Use and Effectiveness of Macroprudential Policies: New Evidence, Journal of Financial Stability, forthcoming.

Chambers, M., C. Garriga, and D. Schlagenhauf, 2009, Accounting for Changes in the Homeownership Rate, International Economic Review 50, 677–726.

Chaney, T., D. Sraer, and D. Thesmar, 2012, The Collateral Channel: How Real Estate Shocks Affect Corporate Investment, American Economic Review 102, 2381–2409.

Christiano, L., R. Motto, and M. Rostagno, 2014, Risk Shocks, American Economic Review 104, 27–65.

Cordoba, J. C. and M. Ripoll, 2004, Credit Cycles Redux, International Economic Review 45, 1011–1046.

Dabla-Norris, E., and N. Srivisal, 2013, Revisiting the Link Between Finance and Macroeconomic Volatility, IMF Working Papers 13/29.

Darbar, S., and X. Wu, 2015, Experiences with Macroprudential Policy, IMF Working Papers 15/123.

De Nicolò, G., and Lucchetta, M., 2013, Systemic Risks and the Macroeconomy, in: NBER Chapters: Quantifying Systemic Risk, Haubrich, J., and A. Lo (eds.), 113–148, University of Chicago Press.

den Haan, W. and V. Sterk, 2010, The Myth of Financial Innovation and the Great Moderation, Economic Journal 121, 707–739.

Denizer, C., M. Iyigun, and A. Owen, 2002, Finance and Macroeconomic Volatility, Contributions to Macroeconomics 2.

Duca, J. V., J. Muellbauer, and A. Murphy, 2011, House Prices and Credit Constraints: Making Sense of the U.S. Experience, Economic Journal 121, 533–551.

Easterly, W., R. Islam, and J. Stiglitz, 2000, Shaken and Stirred: Explaining Growth Volatility, Annual World Bank Conference on Development Economics, World Bank, Washington D.C.

Fair, R. C., and J. B. Taylor, 1983, Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models, Econometrica 51, 1169–1185.

Favilukis, J., S. C. Ludvigson, and S. Van Nieuwerburgh, 2015, The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium, Journal of Political Economy, forthcoming.

Fernández-Villaverde, J., J. F. Rubio-Ramírez, and F. Schorfheide, 2016, Solution and Estimation Methods for DSGE Models, Handbook of Macroeconomics 2, forthcoming.

Gerali, A., S. Neri, L. Sessa, and F. Signoretti, 2010, Credit and Banking in a DSGE Model of the Euro Area, Journal of Money, Credit and Banking 42, 107–141.

Gertler, M., and C. Lown, 1999, The Information in the High-Yield Bond Spread for the Business Cycle: Evidence and Some Implications, Oxford Review of Economic Policy 15, 132–150.

Graham, J. R., M. T. Leary, and M. R. Roberts, 2014, A Century of Capital Structure: The Leveraging of Corporate America, Journal of Financial Economics 118, 658–683.

Guerrieri, L., and M. Iacoviello, 2015, OccBin: A Toolkit for Solving Dynamic Models With Occasionally Binding Constraints Easily, Journal of Monetary Economics 70, 2238.

Guerrieri, L., and M. Iacoviello, 2016, Collateral Constraints and Macroeconomic Asymmetries, mimeo, Federal Reserve Board.

Hamilton, J. D., 1994, Time Series Analysis, Princeton University Press, Princeton, NJ.

Holden, T., and M. Paetz, 2012, Efficient Simulation of DSGE Models with Inequality Constraints, School of Economics Discussion Papers 1612, University of Surrey.

Iacoviello, M., 2005, House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle, American Economic Review 95, 739–764.

Iacoviello, M., 2015, Financial Business Cycles, Review of Economic Dynamics 18, 140–163.

Iacoviello, M. and S. Neri, 2010, Housing Market Spillovers: Evidence from an Estimated DSGE Model, American Economic Journal: Macroeconomics 2, 125–164.

IMF, 2011, Macroprudential Policy: An Organizing Framework, International Monetary Fund, Washington D.C.

Jácome, L., and S. Mitra, 2015, LTV and DTI Limits—Going Granular, IMF Working Papers 15/154.

Jensen, H., I. Petrella, S. H. Ravn, and E. Santoro, 2016, Deepening Contractions and Collateral Constraints, mimeo, University of Copenhagen and Warwick Business School.

Jermann, U. and V. Quadrini, 2009, Financial Innovations and Macroeconomic Volatility, working paper, Universities of Pennsylvania and Southern California.

Jermann, U. and V. Quadrini, 2012, Macroeconomic Effects of Financial Shocks, American Economic Review 102, 238–271.

Jordà, O., M. Schularick, and A. M. Taylor, 2013, When Credit Bites Back, Journal of Money, Credit and Banking 45, 3–28.

Jordà, O., M. Schularick, and A. M. Taylor, 2016, Macrofinancial History and the New Business Cycle Facts, NBER Macroeconomics Annual, forthcoming.

Justiniano, A., G. E. Primiceri, and A. Tambalotti, 2014, Credit Supply and the Housing Boom, working paper, Federal Reserve Bank of Chicago, Northwestern University and Federal Reserve Bank of New York.

Justiniano, A., G. E. Primiceri, and A. Tambalotti, 2015, Household Leveraging and Deleveraging, Review of Economic Dynamics 18, 3–20.

Kiyotaki, N. and J. Moore, 1997, Credit Cycles, Journal of Political Economy 105, 211–248.

Kocherlakota, N. R., 2000, Creating Business Cycles Through Credit Constraints, Quarterly Review, Federal Reserve Bank of Minneapolis, Summer, 2–10.

Laséen, S., and L. E. O. Svensson, 2011, Anticipated Alternative Policy Rate Paths in Policy Simulations, International Journal of Central Banking 7, 1–35.

Lambertini, L., C. Mendicino, and M. T. Punzi, 2013, Leaning Against Boom-Bust Cycles in Credit and Housing Prices, Journal of Economic Dynamics and Control 37, 1500–1522.

Liu, Z., P. Wang, and T. Zha, 2013, Land-Price Dynamics and Macroeconomic Fluctuations, Econometrica 81, 1147–1184.

Maffezzoli, M., and T. Monacelli, 2015, Deleverage and Financial Fragility, CEPR Discussion Papers 10531.

Mallick, D. 2014, Financial Development, Shocks, and Growth Volatility, Macroeconomic Dynamics 18, 651–688.

Mandelman, F. S., P. Rabanal, J. F. Rubio-Ramírez, and D. Vilán, 2011, Investment-Specific Technology Shocks and International Business Cycles: An Empirical Assessment, Review of Economic Dynamics 14, 136–155.

Mendicino, C., 2012a, On the Amplification Roles of Collateral Constraints, Economics Letters 117, 429–435.

Mendicino, C., 2012b, Collateral Requirements: Macroeconomic Fluctuations and Macroprudential Policy, Bank of Portugal Working Papers 11/2012.

Nakov, A., and A. Pescatori, 2010, Oil and the Great Moderation, Economic Journal 120, 131–156.

Ruge-Murcia, Francisco, 2007, Methods to Estimate Dynamic Stochastic General Equilibrium Models, Journal of Economic Dynamics and Control 31, 2599–2636.

Ruge-Murcia, Francisco, 2012, Estimating Nonlinear DSGE Models by the Simulated Method of Moments: With an Application to Business Cycles, Journal of Economic Dynamics and Control 36, 914–938.

Schularick, M., and A. M. Taylor, 2012, Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870–2008, American Economic Review 102, 1029–1061.

Stock, J., and M. Watson, 2003, Has the Business Cycle Changed and Why?, NBER Macroeconomics Annual 2002, 159–230.

Taylor, A. M., 2015, Credit, Financial Stability, and the Macroeconomy, Annual Review of Economics 7, 309–339.

Walentin, K., 2014, Housing Collateral and the Monetary Transmission Mechanism, Scandinavian Journal of Economics 116, 635–668.

Woodford, M., 1986, Stationary Sunspot Equilibria in a Finance Constrained Economy, Journal of Economic Theory, 40, 128–137.

Appendices

A Assets and liabilities in the US

Figure 1 shows the ratio of liabilities to assets for households and firms in the United States, respectively. All data are taken from FRED (Federal Reserve Economic Data), Federal Reserve Bank of St. Louis. The primary source is Flow of Funds data from the Board of Governors of the Federal Reserve System. For business liabilities we use the sum of debt securities and loans of nonfinancial corporate and noncorporate businesses. As assets we follow Liu *et al.* (2013) and use data on both sectors' equipment and software as well as real estate at market value. For households and nonprofit organizations, we again use the sum of debt securities and loans as data for liabilities and use as assets both groups' real estate at market value and equipment and software of nonprofit organizations.[24] For the years 1945–1951, data is only available on an annual basis. For these years, we use linear interpolation to compute quarterly observations.

The ratios reported in Figure 1 are aggregate measures, and may therefore not reflect actual loan-to-value (LTV) requirements for the marginal borrower. Nonetheless, we report these figures since the flow of funds data delivers a continuous measure of LTV ratios covering the entire period 1945–2016. For households, the aggregate ratio of credit to assets in the economy is likely to understate the actual down-payment requirements faced by households applying for a mortgage loan, since loans and assets are not uniformly distributed across households. In our model, we distinguish between patient and impatient households, and we assume that only the latter group is faced with a collateral constraint. In the data, we have not made this distinction, so that the LTV ratio for households reported in Figure 1 represents an average of the LTV of patient households (savers), who are likely to have many assets and small loans, and that of impatient households (borrowers), who on average have larger loans and fewer assets. Justiniano *et al.* (2014) use the Survey of Consumer Finances to make this distinction, and identify borrowers as households with liquid assets of a value less than two months of their income. Based on the surveys from 1992, 1995, and 1998, they arrive at an average LTV ratio for this group of around 0.8, while our measure fluctuates around 0.5 during the 1990s. Another approach, following Duca *et al.* (2011), is to focus on first-time home-buyers, who are likely to fully exploit their borrowing capacity. Using data from the American Housing Survey, these authors report LTV ratios approaching 0.9 towards the end of the 1990s; reaching a peak of almost 0.95 before the onset of the recent crisis. While these alternative approaches are thus likely to result in higher *levels* of LTV ratios, we are interested in the *development over time* of these ratios. While we believe the Flow of Funds data provide the most comprehensive and consistent time series evidence in this respect, substantial increases over time in LTV ratios faced by households have been extensively documented; see, e.g., Campbell and Hercowitz (2009), Duca *et al.* (2011), Favilukis *et al.* (2015), and Boz and Mendoza (2014). It should be noted that for households, various government-sponsored programs directed at lowering the down-payment requirements faced by low-income or first-time home buyers have been enacted by different administrations (Chambers *et al.*, 2009). These are likely to have contributed to the increase in the ratio of loans to assets illustrated in the left panel of Figure 1.

Likewise, the aggregate ratio of business loans to assets in the data may cover for a disparate distribution of credit and assets across firms. In general, the borrowing patterns and conditions of firms are more difficult to characterize than those of households, as their credit demand is more volatile, and their assets less uniform and often more difficult to assess. Liu *et al.* (2013) also use Flow of Funds data to calibrate the LTV ratio of entrepreneurs, and arrive at a value

[24] Until 2015, debt securities and loans were aggregated under the title "Credit market instruments" for businesses as well as households in the Financial Accounts of the United States.

of 0.75. This ratio is based on an assumption that commercial real estate enters with a weight of 0.5 in the asset composition of firms. In contrast, the ratio we report in Figure 1 assigns a weight of 1 to commercial real estate. While the transformation of Liu *et al.* (2013) would result in higher LTV ratios at any point in time, it would not affect the finding of rising LTV ratios over time. The secular increase in firm leverage over the second half of the 20$^{\text{th}}$ century has also been documented by Graham *et al.* (2014) using data from the Compustat database.[25,26] These authors report loan-to-asset ratios that are broadly in line with those we present. More generally, an enhanced access of firms to credit markets over time has been extensively documented in the literature. This involves, for instance, the emergence of a market for high-risk, high-yield bonds (Gertler and Lown, 1999), increased flexibility in firms' financing decisions, and the resulting immoderation in financial quantities (Jermann and Quadrini, 2009).

[25] It should be mentioned that they also show a Flow of Funds-based measure of debt to total assets at historical cost (or book value) for firms. The increase over time in this measure is smaller. However, we believe that the ratio of debt to *pledgeable* assets at market values (as shown in Figure 1) is the relevant measure for firms' access to collateralized loans, and hence more appropriate for our purposes.

[26] We emphasize that Figure 1 reports a *gross* measure of firm leverage. Bates *et al.* (2009) report that firm leverage *net of* cash holdings has been declining since 1980, but that this decline is entirely due to a large increase in cash holdings.

B The steady state

The deterministic steady state of our model is described in the following, where variables without time subscripts are the steady-state values. We first consider the implications of the patient households' optimality conditions. From (6) and (7), we get

$$\frac{1-\beta^P\rho^P}{(1-\rho^P)\,C^P}=\lambda^P, \tag{B.1}$$

and

$$\nu^P\left(1-N^P\right)^{-\varphi^P}=\lambda^P W^P, \tag{B.2}$$

respectively. The steady-state gross interest rate on loans is recovered from (8):

$$\begin{aligned}\beta^P R\lambda^P &= \lambda^P,\\ R &= \frac{1}{\beta^P},\end{aligned} \tag{B.3}$$

emphasizing that it is the time preferences of the most patient individual that determine the steady-state rate of interest. From (9) we find

$$\begin{aligned}\frac{\varepsilon}{H^P}+\beta^P\lambda^P Q &= \lambda^P Q,\\ H^P &= \frac{\varepsilon}{Q\lambda^P\left(1-\beta^P\right)}.\end{aligned} \tag{B.4}$$

Turning to the impatient households, (10) and (11), leads to

$$\frac{1-\beta^I\rho^I}{(1-\rho^I)\,C^I}=\lambda^I, \tag{B.5}$$

and

$$\nu^I\left(1-N^I\right)^{-\varphi^I}=\lambda^I W^I, \tag{B.6}$$

respectively. From (12) we obtain the steady-state value of the multiplier on the credit constraint:

$$\mu^I=\lambda^I\left(1-\beta^I R\right),$$

which by use of (B.3) yields

$$\mu^I=\lambda^I\left(1-\frac{\beta^I}{\beta^P}\right). \tag{B.7}$$

From (B.7), we see that in steady state $\mu^I>0$ since $\beta^P>\beta^I$, which proves that the credit constraint (4) is binding in steady state. In a similar fashion, we get from (20):

$$\mu^E=\lambda^E\left(1-\frac{\beta^E}{\beta^P}\right). \tag{B.8}$$

Hence, $\mu^E > 0$ implying that the entrepreneurs' credit constraints, (18), are also binding in steady state. From (13) we get

$$\begin{aligned}
\frac{\varepsilon}{H^I} + \beta^I \lambda^I Q + \mu^I s \frac{Q}{R} &= \lambda^I Q, \\
H^I &= \frac{\varepsilon}{Q\lambda^I \left[1 - \beta^I - \frac{\mu^I}{\lambda^I} s \frac{1}{R}\right]}, \\
H^I &= \frac{\varepsilon}{Q\lambda^I \left[1 - \beta^I - \left(1 - \frac{\beta^I}{\beta^P}\right) s \beta^P\right]}, \\
H^I &= \frac{\varepsilon}{Q\lambda^I \left[1 - \beta^I - \left(\beta^P - \beta^I\right) s\right]},
\end{aligned} \tag{B.9}$$

where the next-to-last line makes use of (B.3) and (B.7).

Turning to the remaining optimality conditions of the entrepreneurs, (19) gives

$$\frac{1 - \beta^E \rho^E}{\left(1 - \rho^E\right) C^E} = \lambda^E, \tag{B.10}$$

and (21) implies

$$\psi^E \left[1 - \frac{\Omega}{2}\left(\frac{I}{I} - 1\right)^2\right] - \psi^E \Omega \frac{I}{I}\left(\frac{I}{I} - 1\right) + \beta^E \psi^E \Omega \left(\frac{I}{I}\right)^2 \left(\frac{I}{I} - 1\right) = \lambda^E$$

leading to

$$\psi^E = \lambda^E. \tag{B.11}$$

This reflects that there are no investment adjustment costs in steady state in our no-growth model. Therefore, the shadow value of a unit of capital equals the shadow value of wealth. Combining this with (25), we readily obtain

$$Q^K = 1. \tag{B.12}$$

From (22) we obtain

$$\begin{aligned}
\beta^E r^K \lambda^E + \beta^E (1 - \delta) \psi^E + \mu^E s \frac{Q^K}{R} &= \psi^E \\
\beta^E \lambda^E r^K + \beta^E (1 - \delta) \psi^E + \lambda^E \left(1 - \frac{\beta^E}{\beta^P}\right) s \frac{Q^K}{R} &= \psi^E \\
1 + r^K - \delta &= \frac{1 - \left(\beta^P - \beta^E\right) s Q^K}{\beta^E},
\end{aligned} \tag{B.13}$$

where the second line uses (B.8), and the last uses (B.3) and (B.11), respectively. From (23) we find:

$$r^H = \frac{\left(1 - \beta^E\right) Q}{\beta^E} - \frac{\mu^E s}{\lambda^E \beta^E} \frac{Q}{R}. \tag{B.14}$$

We then turn to the remaining equilibrium conditions in steady state. As we saw above, the two credit constraints are binding in steady state. Hence,

$$B^I = \frac{sQH^I}{R}, \tag{B.15}$$

$$B^E = s\frac{Q^K K + QH^E}{R}. \tag{B.16}$$

The production function is

$$Y = \left[\left(N^P\right)^{\alpha}\left(N^I\right)^{1-\alpha}\right]^{\gamma}\left[\left(H^E\right)^{\phi} K^{1-\phi}\right]^{1-\gamma}. \tag{B.17}$$

The steady-state versions of the firms' first-order conditions taking market clearing conditions into account, (28)–(31), are

$$\begin{aligned}
\alpha\gamma\frac{Y}{N^P} &= W^P, &(B.18)\\
(1-\alpha)\,\gamma\frac{Y}{N^I} &= W^I, &(B.19)\\
(1-\gamma)\,(1-\phi)\,\frac{Y}{K} &= r^K, &(B.20)\\
(1-\gamma)\,\phi\frac{Y}{H^E} &= r^H. &(B.21)
\end{aligned}$$

In steady state, the law of motion for capital implies

$$I = \delta K. \tag{B.22}$$

We have the following steady-state resource constraints:

$$Y = C^P + C^I + C^E + I, \tag{B.23}$$

$$H = H^P + H^I + H^E, \tag{B.24}$$

$$B^P + B^I + B^E = 0. \tag{B.25}$$

Also, we have the steady-state versions of the agents' budget constraints:

$$\begin{aligned}
C^P &= W^P N^P - (R-1)\,B^P, &(B.26)\\
C^I &= W^I N^I - (R-1)\,B^I, &(B.27)\\
C^E + I &= r^K K + r^H H^E - (R-1)\,B^E &(B.28)
\end{aligned}$$

(One of these is redundant by Walras' law.)

We therefore have that the steady state is characterized by the vector

$$\left[\begin{array}{c} Y, C^P, C^I, C^E, I, H^P, H^I, H^E, K, N^P, N^I, B^P, B^I, B^E, \\ Q, Q^K, R, r^K, r^H, W^P, W^I, \lambda^P, \lambda^I, \lambda^E, \mu^I, \mu^E, \psi^E \end{array}\right].$$

These 27 variables are determined by the 27 equations (B.1), (B.2), (B.3), (B.4), (B.5), (B.6), (B.7), (B.8), (B.9), (B.10), (B.11), (B.13), (B.14), (B.15), (B.16), (B.17), (B.18), (B.19), (B.20), (B.21), (B.22), (B.23), (B.24), (B.25), (B.26), (B.27), and (B.12).

We now briefly proceed with a characterization of the steady state, wherein we compute some variables in ratios to output in closed form. Then we reduce the system to one of seven equations in central quantities, which is solved numerically, conditional on these ratios. The remaining 19 variables then follow explicitly from the characterizations given above. First, combine (B.20) and (B.13) to get an expression for capital-output ratio:

$$\frac{K}{Y} = \frac{\beta^E\,(1-\gamma)\,(1-\phi)}{1-\left(\beta^P-\beta^E\right)s-\beta^E\,(1-\delta)}, \tag{B.29}$$

where we have used that $Q^K = 1$ by (B.12), Then we combine (B.14) and (B.21) to get an expression for entrepreneurs' housing-output ratio:

$$\begin{aligned}
(1-\gamma)\,\phi \frac{Y}{H^E} &= \frac{\left(1-\beta^E\right)Q}{\beta^E} - \frac{\mu^E s}{\lambda^E \beta^E}\frac{Q}{R}, \\
\frac{Y}{H^E} &= \frac{\left(1-\beta^E\right)Q\lambda^E R - \mu^E s Q}{(1-\gamma)\,\phi\beta^E\lambda^E R}, \\
\frac{QH^E}{Y} &= \frac{(1-\gamma)\,\phi\beta^E}{\left(1-\beta^E\right) - \left(\beta^P - \beta^E\right)s},
\end{aligned} \tag{B.30}$$

where the last line uses (B.8). Again using that $Q^K = 1$, the borrowing constraint for entrepreneurs (B.16) can be rewritten in terms of ratios to output as

$$\frac{B^E}{Y} = \frac{s}{R}\left(\frac{K}{Y} + \frac{QH^E}{Y}\right),$$

which by use of (B.29), (B.30) and (B.3) imply

$$\frac{B^E}{Y} = \beta^P s\left(\frac{\beta^E(1-\gamma)(1-\phi)}{1-\left(\beta^P-\beta^E\right)s - \beta^E(1-\delta)} + \frac{(1-\gamma)\,\phi\beta^E}{\left(1-\beta^E\right) - \left(\beta^P-\beta^E\right)s}\right) \tag{B.31}$$

This closed-form solution of the entrepreneurs' steady-state loan-to-output ratio is central in setting up a subsystem of seven central variables. First, it can be used with the entrepreneur's budget constraint, (B.28), in ratio to output:

$$\frac{C^E}{Y} + \frac{I}{Y} = r^K\frac{K}{Y} + r^H\frac{H^E}{Y} - (R-1)\frac{B^E}{Y},$$

which by use of (B.22) becomes

$$\frac{C^E}{Y} = \left(r^K - \delta\right)\frac{K}{Y} + r^H\frac{H^E}{Y} - (R-1)\frac{B^E}{Y}.$$

Using (B.13) and (B.21) we get

$$\frac{C^E}{Y} = \left(\frac{1-\beta^E-\left(\beta^P-\beta^E\right)s}{\beta^E}\right)\frac{K}{Y} + (1-\gamma)\,\phi - (R-1)\frac{B^E}{Y},$$

which by use of (B.29) provides the entrepreneurs' consumption-to-output ratio:

$$\frac{C^E}{Y} = \frac{(1-\gamma)(1-\phi)\left[1-\beta^E-\left(\beta^P-\beta^E\right)s\right]}{1-\left(\beta^P-\beta^E\right)s-\beta^E(1-\delta)} + (1-\gamma)\,\phi - \frac{1-\beta^P}{\beta^P}\frac{B^E}{Y}, \tag{B.32}$$

Then turn to the impatient households. In ratio to output, their budget constraints are, cf. (B.27),

$$\frac{C^I}{Y} = \frac{W^I N^I}{Y} - (R-1)\frac{B^I}{Y},$$

which by use of (B.19) and (B.3) becomes

$$\frac{C^I}{Y} = (1-\alpha)\,\gamma - \frac{1-\beta^P}{\beta^P}\frac{B^I}{Y}.$$

Likewise, the patient households' budget constraints are written as, cf. (B.26),

$$\frac{C^P}{Y} = \frac{W^P N^P}{Y} - (R-1)\frac{B^P}{Y},$$

which by use of (B.18) and (B.3) becomes

$$\frac{C^P}{Y} = \alpha\gamma - \frac{1-\beta^P}{\beta^P}\frac{B^P}{Y}.$$

Adding these constraints gives

$$\frac{C^I + C^P}{Y} = \gamma + \frac{1-\beta^P}{\beta^P}\frac{B^E}{Y}, \tag{B.33}$$

where (B.25) has been invoked. Note that the right-hand-side of (B.33) is known by (B.31).

Combining (B.1), (B.2) and (B.18) gives the steady-state equilibrium condition for the labor market for patient households:

$$\nu^P\left(1-N^P\right)^{-\varphi^P} C^P \frac{1-\rho^P}{1-\beta^P\rho^P} = \alpha\gamma\frac{Y}{N^P}. \tag{B.34}$$

Similarly, (B.5), (B.6) and (B.19) characterize the labor-market equilibrium for impatient households:

$$\nu^I\left(1-N^I\right)^{-\varphi^I} C^I \frac{1-\rho^I}{1-\beta^I\rho^I} = (1-\alpha)\gamma\frac{Y}{N^I}. \tag{B.35}$$

Combining the two households' land demand expressions, (B.4) and (B.9), gives

$$\frac{H^I}{H^P} = \frac{\lambda^P\left(1-\beta^P\right)}{\lambda^I\left[1-\beta^I-\left(\beta^P-\beta^I\right)s\right]}.$$

Eliminating the multipliers by (B.1) and (B.5), and eliminating H^P by (B.24), we obtain the following land-market equilibrium characterization:

$$\begin{aligned}
\frac{H^I}{H-H^I-H^E} &= \frac{\frac{1-\beta^P\rho^P}{(1-\rho^P)C^P}\left(1-\beta^P\right)}{\frac{1-\beta^I\rho^I}{(1-\rho^I)C^I}\left[1-\beta^I-\left(\beta^P-\beta^I\right)s\right]},\\
\frac{H^I}{H-H^I-H^E}\frac{C^P}{C^I} &= \frac{\left(1-\beta^P\rho^P\right)\left(1-\rho^I\right)}{\left(1-\rho^P\right)\left(1-\beta^I\rho^I\right)}\frac{\left(1-\beta^P\right)}{\left[1-\beta^I-\left(\beta^P-\beta^I\right)s\right]}.
\end{aligned} \tag{B.36}$$

We also take the impatient households' borrowing constraint into consideration. Using (B.15) to eliminate B^I in the budget constraint, it becomes

$$\begin{aligned}
\frac{C^I}{Y} &= (1-\alpha)\gamma - \frac{1-\beta^P}{\beta^P}\frac{sQH^I}{YR},\\
&= (1-\alpha)\gamma - \left(1-\beta^P\right)\frac{sQH^I}{Y}.
\end{aligned} \tag{B.37}$$

We can use that (B.9) implies

$$QH^I = \frac{\varepsilon}{\lambda^I\left[1-\beta^I-\left(\beta^P-\beta^I\right)s\right]}$$

and thus, again using (B.5),

$$QH^I = \frac{\frac{\varepsilon\left(1-\rho^I\right)}{1-\beta^I\rho^I}C^I}{1-\beta^I-\left(\beta^P-\beta^I\right)s}, \tag{B.38}$$

$$Q = \frac{\frac{\varepsilon\left(1-\rho^I\right)}{1-\beta^I\rho^I}C^I}{H^I\left[1-\beta^I-\left(\beta^P-\beta^I\right)s\right]}. \tag{B.39}$$

We then use (B.38) to rewrite the consumption-output ratio for impatient households (B.37) as:

$$\begin{aligned}\frac{C^I}{Y} &= (1-\alpha)\gamma-\left(1-\beta^P\right)\frac{sQH^I}{Y} \\ &= (1-\alpha)\gamma-\left(1-\beta^P\right)\frac{s}{Y}\frac{\frac{\varepsilon\left(1-\rho^I\right)}{1-\beta^I\rho^I}C^I}{1-\beta^I-\left(\beta^P-\beta^I\right)s}.\end{aligned} \tag{B.40}$$

Likewise, we rewrite the entrepreneurs' land to output ratio by using (B.39) to eliminate Q from (B.30):

$$\frac{H^E}{Y} = \frac{(1-\gamma)\phi\beta^E}{1-\beta^E-\left(\beta^P-\beta^E\right)s}\frac{H^I\left[1-\beta^I-\left(\beta^P-\beta^I\right)s\right]}{\frac{\varepsilon(1-\rho^I)}{1-\beta^I\rho^I}C^I}. \tag{B.41}$$

Finally, the production function (B.17) is rewritten as a function of the derived ratios:

$$Y^\gamma = A\left[\left(N^P\right)^\alpha\left(N^I\right)^{1-\alpha}\right]^\gamma\left[\left(\frac{H^E}{Y}\right)^\phi\left(\frac{K}{Y}\right)^{1-\phi}\right]^{1-\gamma},$$

Using (B.29), we finally obtain

$$Y = A^{\frac{1}{\gamma}}\left(N^P\right)^\alpha\left(N^I\right)^{1-\alpha}\left[\left(\frac{H^E}{Y}\right)^\phi\left(\frac{\beta^E(1-\gamma)(1-\phi)}{1-\left(\beta^P-\beta^E\right)s-\beta^E(1-\delta)}\right)^{1-\phi}\right]^{\frac{1-\gamma}{\gamma}}. \tag{B.42}$$

We have now reduced the steady-state to a matter of finding the vector

$$\left[Y, C^P, C^I, H^I, H^E, N^P, N^I\right],$$

which satisfies the equations (B.33), (B.34), (B.35), (B.36), (B.40), (B.41) and (B.42), *given* the solution for B^E/Y, (B.31), and given all parameters and exogenous variables of the model. We compute the vector numerically using `fsolve` in MATLAB. The remaining 19 variables then follow analytically from the steady-state equations presented above.

C The log-linearized model

We log-linearize the model around the steady state found in the previous section. In the following, we let $\widehat{X_t}$ denote the log-deviation of a generic variable X_t from its steady state value X, except for the following variables. For the interest rates, $\widehat{R}_t \equiv R_t - R$, $\widehat{r}_t^H \equiv r_t^H - r^H$ and $\widehat{r}_t^K \equiv r_t^K - r^K$, and for debt, $\widehat{B}_t^i \equiv \left(B_t^i - B^i\right)/Y$, $i = P, I, E$. We first derive the log-linear versions of the agents' optimality conditions and conclude with the expressions for market clearing.

C.1 Optimality conditions of the patient households

Equations (6), (7) and (8) readily becomes

$$\beta^P \rho^P \mathrm{E}_t \left\{\widehat{C}_{t+1}^P\right\} - \left(1 + \beta^P \left(\rho^P\right)^2\right) \widehat{C}_t^P + \rho^P \widehat{C}_{t-1}^P = \left(1 - \rho^P\right)\left(1 - \beta^P \rho^P\right) \widehat{\lambda}_t^P, \tag{C.1}$$

$$\varphi^P \frac{N^P}{1 - N^P} \widehat{N}_t^P = \widehat{\lambda}_t^P + \widehat{W}_t^P \tag{C.2}$$

$$\beta^P \widehat{R}_t + \mathrm{E}_t \left\{\widehat{\lambda}_{t+1}^P\right\} = \widehat{\lambda}_t^P, \tag{C.3}$$

Log-linearization of (9) yields

$$\frac{\varepsilon}{H^P}\left(\widehat{\varepsilon}_t - \widehat{H}_t^P\right) + \beta^P \lambda^P Q \mathrm{E}_t \left\{\widehat{\lambda}_{t+1}^P + \widehat{Q}_{t+1}\right\} = \lambda^P Q \left(\widehat{\lambda}_t^P + \widehat{Q}_t\right).$$

Now use steady-state equation (B.4) to get

$$-Q\lambda^P \left(1 - \beta^P\right) \widehat{H}_t^P + Q\lambda^P \left(1 - \beta^P\right) \widehat{\varepsilon}_t + \beta^P \lambda^P Q \mathrm{E}_t \left\{\widehat{\lambda}_{t+1}^P + \widehat{Q}_{t+1}\right\} = \lambda^P Q \left(\widehat{\lambda}_t^P + \widehat{Q}_t\right),$$

and thereby

$$\beta^P \mathrm{E}_t \left\{\widehat{\lambda}_{t+1}^P + \widehat{Q}_{t+1}\right\} - \left(1 - \beta^P\right) \widehat{H}_t^P + \left(1 - \beta^P\right) \widehat{\varepsilon}_t = \widehat{\lambda}_t^P + \widehat{Q}_t. \tag{C.4}$$

Moreover, the log-linearized budget constraint holds:

$$\begin{aligned} & \frac{C^P}{Y} \widehat{C}_t^P + \frac{QH^P}{Y}\left(\widehat{H}_t^P - \widehat{H}_{t-1}^P\right) + \frac{B^P}{Y} \widehat{R}_{t-1} + \frac{1}{\beta^P} \widehat{B}_{t-1}^P \\ = \; & \widehat{B}_t^P + \alpha\gamma\left(\widehat{W}_t^P + \widehat{N}_t^P\right). \end{aligned}$$

where we have used (B.18). This constraint, however, does not feature in our Matlab codes (we use the impatient households' and entrepreneurs' budget constraint and the economy-wide resource constraint).

C.2 Optimality conditions of the impatient households

From (10), (11) and (12) we obtain

$$\beta^I \rho^I \mathrm{E}_t \left\{\widehat{C}_{t+1}^I\right\} - \left(1 + \beta^I \left(\rho^I\right)^2\right) \widehat{C}_t^I + \rho^I \widehat{C}_{t-1}^I = \left(1 - \rho^I\right)\left(1 - \beta^I \rho^I\right) \widehat{\lambda}_t^I, \tag{C.5}$$

$$\varphi^I \frac{N^I}{1 - N^I} \widehat{N}_t^I = \widehat{\lambda}_t^I + \widehat{W}_t^I, \tag{C.6}$$

and

$$\beta^I \lambda^I \widehat{R}_t + \beta^I R \lambda^I \mathrm{E}_t \left\{\widehat{\lambda}_{t+1}^I\right\} + \mu^I \widehat{\mu}_t^I = \lambda^I \widehat{\lambda}_t^I,$$

respectively. The last expression is rewritten by use of (B.7):

$$\beta^I \widehat{R}_t + \beta^I R \mathrm{E}_t \left\{ \widehat{\lambda}^I_{t+1} \right\} + \left(1 - \frac{\beta^I}{\beta^P} \right) \widehat{\mu}^I_t = \widehat{\lambda}^I_t. \tag{C.7}$$

Furthermore, (13) becomes

$$\begin{aligned} & \frac{\varepsilon}{H^I} \left(\widehat{\varepsilon}_t - \widehat{H}^I_t \right) + \beta^I \lambda^I Q \mathrm{E}_t \left\{ \widehat{\lambda}^I_{t+1} + \widehat{Q}_{t+1} \right\} \\ & + \mu^I \frac{sQ}{R} \left[\widehat{\mu}^I_t + \widehat{s}_t + \mathrm{E}_t \left\{ \widehat{Q}_{t+1} \right\} - \beta^P \widehat{R}_t \right] \\ = \; & \lambda^I Q \left(\widehat{\lambda}^I_t + \widehat{Q}_t \right), \end{aligned}$$

which by use of (B.7) and (B.9) becomes

$$\begin{aligned} & Q \lambda^I \left[1 - \beta^I - \left(\beta^P - \beta^I \right) s \right] \left(\widehat{\varepsilon}_t - \widehat{H}^I_t \right) + \beta^I \lambda^I Q \mathrm{E}_t \left\{ \widehat{\lambda}^I_{t+1} + \widehat{Q}_{t+1} \right\} \\ & + \lambda^I \left(1 - \frac{\beta^I}{\beta^P} \right) \frac{sQ}{R} \left[\widehat{\mu}^I_t + \widehat{s}_t + \mathrm{E}_t \left\{ \widehat{Q}_{t+1} \right\} - \beta^P \widehat{R}_t \right] \\ = \; & \lambda^I Q \left(\widehat{\lambda}^I_t + \widehat{Q}_t \right), \end{aligned}$$

$$\begin{aligned} & \beta^I \mathrm{E}_t \left\{ \widehat{\lambda}^I_{t+1} + \widehat{Q}_{t+1} \right\} - \left[1 - \beta^I - s \left(\beta^P - \beta^I \right) \right] \widehat{H}^I_t \\ & + \left[1 - \beta^I - \left(\beta^P - \beta^I \right) s \right] \widehat{\varepsilon}_t \\ & + s \left(\beta^P - \beta^I \right) \left[\widehat{\mu}^I_t + \widehat{s}_t + \mathrm{E}_t \left\{ \widehat{Q}_{t+1} \right\} - \beta^P \widehat{R}_t \right] \\ = \; & \widehat{\lambda}^I_t + \widehat{Q}_t. \end{aligned} \tag{C.8}$$

where we have again used (B.3). The budget constraint becomes

$$\frac{C^I}{Y} \widehat{C}^I_t + \frac{QH^I}{Y} \left(\widehat{H}^I_t - \widehat{H}^I_{t-1} \right) + \frac{B^I}{Y} \widehat{R}_{t-1} + \frac{1}{\beta^P} \widehat{B}^I_{t-1} = \widehat{B}^I_t + (1-\alpha) \gamma \left(\widehat{W}^I_t + \widehat{N}^I_t \right), \tag{C.9}$$

where we have used (B.19). Finally, the log-linearized version of (4) holds:

$$\frac{Y}{B^I} \widehat{B}^I_t \leq \hat{s}_t + \mathrm{E}_t \left\{ \widehat{Q}_{t+1} \right\} + \widehat{H}^I_t - \beta^P \widehat{R}_t. \tag{C.10}$$

Note that while the credit constraint binds in steady state, cf. (B.15), we allow it to be non-binding outside steady state.

C.3 Optimality conditions of the entrepreneurs

From (B.8) and (20) we get

$$\beta^E \rho^E \mathrm{E}_t \left\{ \widehat{C}^E_{t+1} \right\} - \left(1 + \beta^E \left(\rho^E \right)^2 \right) \widehat{C}^E_t + \rho^E \widehat{C}^E_{t-1} = \left(1 - \rho^E \right) \left(1 - \beta^E \rho^E \right) \widehat{\lambda}^E_t, \tag{C.11}$$

$$\beta^E \widehat{R}_t + \beta^E R \mathrm{E}_t \left\{ \widehat{\lambda}^E_{t+1} \right\} + \left(1 - \frac{\beta^E}{\beta^P} \right) \widehat{\mu}^E_t = \widehat{\lambda}^E_t. \tag{C.12}$$

From (21) we derive

$$\widehat{\psi}_t^E - \Omega\left(1+\beta^E\right)\widehat{I}_t + \Omega\widehat{I}_{t-1} + \beta^E\Omega\mathrm{E}_t\left\{\widehat{I}_{t+1}\right\} = \widehat{\lambda}_t^E, \tag{C.13}$$

where we have made use of (B.11). Equation (22) becomes

$$\begin{aligned} & \beta^E\widehat{r}_t^K + \beta^E r^K\mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^E\right\} + (1-\delta)\,\beta^E\mathrm{E}_t\left\{\widehat{\psi}_{t+1}^E\right\} \\ & + \left(\beta^P-\beta^E\right)sQ^K\left(\widehat{\mu}_t^E + \widehat{s}_t + \mathrm{E}_t\left\{\widehat{Q}_{t+1}^K\right\} - \beta^P\widehat{R}_t\right) \\ = \ & \widehat{\psi}_t^E \end{aligned} \tag{C.14}$$

where we have used (B.11) and (B.3). Moreover, (25) becomes

$$\widehat{\psi}_t^E = \widehat{\lambda}_t^E + \widehat{Q}_t^K. \tag{C.15}$$

Finally, (23) is approximated as

$$\begin{aligned} & \beta^E Q\left(\mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^E\right\} + \mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\}\right) + \beta^E r^H\left(\mathrm{E}_t\left\{\widehat{\lambda}_{t+1}^E\right\} + \frac{1}{r^H}\widehat{r}_t^H\right) \\ & + \left(\beta^P-\beta^E\right)sQ\left(\widehat{\mu}_t^E + \widehat{s}_t + \mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} - \beta^P\widehat{R}_t\right) \\ = \ & Q\left(\widehat{\lambda}_t^E + \widehat{Q}_t\right). \end{aligned} \tag{C.16}$$

Furthermore, the budget constraint for entrepreneurs becomes

$$\begin{aligned} & \frac{C^E}{Y}\widehat{C}_t^E + \frac{I}{Y}\widehat{I}_t + \frac{QH^E}{Y}\left(\widehat{H}_t^E - \widehat{H}_{t-1}^E\right) + \frac{B^E}{Y}\widehat{R}_{t-1} + \frac{1}{\beta^P}\widehat{B}_{t-1}^E \\ = \ & \widehat{B}_t^E + \frac{K}{Y}\widehat{r}_{t-1}^K + \frac{H^E}{Y}\widehat{r}_{t-1}^H + (1-\gamma)\,\phi\widehat{H}_{t-1}^E + (1-\gamma)\,(1-\phi)\,\widehat{K}_{t-1}. \end{aligned} \tag{C.17}$$

where we have used (B.20) and (B.21). The borrowing constraint must be satisfied:

$$\begin{aligned} Y\widehat{B}_t^E \ \leq \ & s\frac{\left(K+QH^E\right)}{R}\widehat{s}_t - \frac{s}{R^2}\left(K+QH^E\right)\widehat{R}_t + \frac{sK}{R}\mathrm{E}_t\left\{\widehat{Q}_{t+1}^K\right\} \\ & + \frac{sK}{R}\widehat{K}_t + \frac{sQH^E}{R}\mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} + \frac{sQH^E}{R}\widehat{H}_t^E. \end{aligned}$$

Dividing by the steady-state values on both sides:

$$\begin{aligned} \frac{Y}{B^E}\widehat{B}_t^E \ \leq \ & \widehat{s}_t - \beta^P\widehat{R}_t \\ & + \frac{K}{K+QH^E}\mathrm{E}_t\left\{\widehat{Q}_{t+1}^K\right\} + \frac{K}{K+QH^E}\widehat{K}_t + \frac{QH^E}{K+QH^E}\mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} + \frac{QH^E}{K+QH^E}\widehat{H}_t^E, \end{aligned}$$

yielding

$$\frac{Y}{B^E}\widehat{B}_t^E \leq \widehat{s}_t - \beta^P\widehat{R}_t + \frac{K}{K+QH^E}\left(\mathrm{E}_t\left\{\widehat{Q}_{t+1}^K\right\} + \widehat{K}_t\right) + \frac{QH^E}{K+QH^E}\left(\mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} + \widehat{H}_t^E\right). \tag{C.18}$$

C.4 Optimality conditions of the firms

The first-order conditions of firms, (28), (29), (30) and (31), are readily rewritten as

$$\widehat{Y}_t - \widehat{N}_t^P = \widehat{W}_t^P, \tag{C.19}$$

$$\widehat{Y}_t - \widehat{N}_t^I = \widehat{W}_t^I, \tag{C.20}$$

$$\mathrm{E}_t\left\{\widehat{Y}_{t+1}\right\} - \widehat{K}_t = \left(r^K\right)^{-1}\widehat{r}_t^K, \tag{C.21}$$

$$\mathrm{E}_t\left\{\widehat{Y}_{t+1}\right\} - \widehat{H}_t^E = \left(r^H\right)^{-1}\widehat{r}_t^H, \tag{C.22}$$

respectively.

C.5 Market clearing and resource constraints

From the law of motion for capital, (17), we get:

$$\widehat{K}_t = (1-\delta)\,\widehat{K}_{t-1} + \delta\widehat{I}_t. \tag{C.23}$$

where we have used (B.22). Moreover, from the resource constraint, (34), we have:

$$\widehat{Y}_t = \frac{C^P}{Y}\widehat{C}_t^P + \frac{C^I}{Y}\widehat{C}_t^I + \frac{C^E}{Y}\widehat{C}_t^E + \delta\frac{K}{Y}\widehat{I}_t \tag{C.24}$$

We also have the linearized versions of (26), (32) and (33):

$$\widehat{Y}_t = \widehat{A}_t + \alpha\gamma\widehat{N}_t^P + (1-\alpha)\,\gamma\widehat{N}_t^I + (1-\gamma)\,(1-\phi)\,\widehat{K}_{t-1} + (1-\gamma)\,\phi\widehat{H}_{t-1}^E \tag{C.25}$$

$$0 = H^P\widehat{H}_t^P + H^I\widehat{H}_t^I + H^E\widehat{H}_t^E \tag{C.26}$$

$$0 = \widehat{B}_t^P + \widehat{B}_t^I + \widehat{B}_t^E \tag{C.27}$$

Finally, we have the shock processes. For the technology shock, we have from (27):

$$\widehat{A}_t = \rho_A\widehat{A}_{t-1} + z_t. \tag{C.28}$$

Furthermore, we have from (2):

$$\widehat{\varepsilon}_t = \rho_\varepsilon\widehat{\varepsilon}_{t-1} + u_t \tag{C.29}$$

Finally, we have from (5) that

$$\widehat{s}_t = \rho_s\widehat{s}_{t-1} + v_t, \tag{C.30}$$

which completes our list of log-linearized equations.

The log-linearized system consists of 30 equations: 18 first-order conditions, 2 budget constraints, 2 credit constraints, 1 production function, 3 market clearing conditions, 1 capital accumulation equation, and 3 shock processes. The 30 variables of the system are given by the vector

$$\left[\begin{array}{c} \widehat{C}_t^P, \widehat{C}_t^I, \widehat{C}_t^E, \widehat{\lambda}_t^P, \widehat{\lambda}_t^I, \widehat{\lambda}_t^E, \widehat{\psi}_t^E, \widehat{\mu}_t^I, \widehat{\mu}_t^E, \widehat{R}_t, \widehat{N}_t^P, \widehat{N}_t^I, \widehat{W}_t^P, \widehat{W}_t^I, \\ \widehat{H}_t^P, \widehat{H}_t^I, \widehat{H}_t^E, \widehat{Q}_t, \widehat{Q}_t^K, \widehat{r}_t^H, \widehat{r}_t^K, \widehat{K}_t, \widehat{I}_t, \widehat{Y}_t, \widehat{B}_t^P, \widehat{B}_t^I, \widehat{B}_t^E, \widehat{A}_t, \widehat{\varepsilon}_t, \widehat{s}_t \end{array}\right],$$

and is characterized by equations (C.1)-(C.30).

D The solution method

As discussed in the main text, we treat the collateral constraints as inequalities when we solve the model, and add two complementary slackness conditions; (14) and (24), respectively. We then adopt the solution method of Holden and Paetz (2012), on which this appendix builds. In turn, Holden and Paetz (2012) expand on previous work by Laséen and Svensson (2011). With first-order perturbations, this solution method is equivalent to the piecewise linear approach developed by Guerrieri and Iacoviello (2015), as discussed by these authors and confirmed by our own numerical exercises.[27] Finally, Holden and Paetz (2012) and Guerrieri and Iacoviello (2015) evaluate the accuracy of their respective methods against a global solution based on projection methods. This is done for a very simple model with a borrowing constraint, for which a highly accurate global solution can be obtained and used as a benchmark. They find that the non-linear local approximations are very accurate. For the model used in this paper, the large number of state variables (14 endogenous state variables and 3 shocks) renders the use of global solution methods impractical due to the curse of dimensionality typically associated with such methods.

The collateral constraints put an upper bound on the borrowing of each of the two constrained agents. While the constraints are binding in the steady state, this may not be the case outside the steady state, where the constraints may be occasionally binding. Observe that we can reformulate the collateral constraints in terms of restrictions on each agent's shadow value of borrowing; μ_t^i, $i = \{I, E\}$: We know that $\mu_t^i \geq 0$ if and only if the optimal debt level of agent i is exactly at or above the credit limit. In other words, we need to ensure that $\mu_t^i \geq 0$. If this restriction is satisfied with inequality, the constraint is binding, so the slackness condition is satisfied. If it holds with equality, the collateral constraint becomes non-binding, but the slackness condition is still satisfied. If instead $\mu_t^i < 0$, agent i's optimal level of debt is lower than the credit limit, so that treating his collateral constraint as an equality implies that we are forcing him to borrow "too much." In this case, the slackness condition is violated. We then need to add shadow price shocks so as to "push" μ_t^i back up until it exactly equals its lower limit of zero and the slackness condition is satisfied. The idea of adding such shocks to the model derives from Laséen and Svensson (2011), who use such an approach to deal with pre-announced paths for the interest rate setting of a central bank. The contribution of Holden and Paetz (2012) is to develop a numerical method to compute the size of these shocks that are required to obtain the desired level for a given variable in each period, and to make this method applicable to a general class of potentially more complicated problems than the relatively simple experiments conducted by Laséen and Svensson (2011).

We first describe how to compute impulse responses to a single shock, e.g., a technology shock. The first step is to add independent sets of shadow price shocks to each of the two log-linearized collateral constraints. To this end, we need to determine the number of periods T in which we conjecture that the collateral constraints may be non-binding. This number may be smaller than or equal to the number of periods for which we compute impulse responses; $T \leq T^{IRF}$. For each period $t \leq T$, we then add shadow price shocks which hit the economy in period t but become known at period 0, that is, at the same time the economy is hit by the technology shock. In other words, the log-linearized collateral constraints become:

$$\frac{Y}{B^I}\widehat{B}_t^I = \widehat{s}_t + \mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} + \widehat{H}_t^I - \beta^P\widehat{R}_t - \sum_{j=0}^{T-1}\varepsilon_{j,t-j}^{SP,I},$$

[27] These authors also emphasize the equivalence between these two approaches and the widely used extended path algorithm of Fair and Taylor (1983).

$$\frac{Y}{B^E}\widehat{B}_t^E = \widehat{s}_t - \beta^P\widehat{R}_t + \frac{K}{K+QH^E}\left(\mathrm{E}_t\left\{\widehat{Q}_{t+1}^K\right\} + \widehat{K}_t\right) + \frac{QH^E}{K+QH^E}\left(\mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} + \widehat{H}_t^E\right) - \sum_{j=0}^{T-1}\varepsilon_{j,t-j}^{SP,E},$$

where $\varepsilon_{j,t-j}^{SP,i}$ is the shadow price shock that hits agent i in period $t=j$, and is anticipated by all agents in period $t=t-j=0$ ensuring consistency with rational expectations. We let all shadow price shocks be of unit magnitude. We then need to compute two sets of weights α_{μ_I} and α_{μ_E} to control the impact of each shock on μ_t^I and μ_t^E. The "optimal" sets of weights ensure that μ_t^I and μ_t^E are bounded below at exactly zero. The weights are computed by solving the following quadratic programming problem:

$$\begin{aligned}\alpha^* &\equiv \begin{bmatrix}\alpha_{\mu_I}^{*\prime} & \alpha_{\mu_E}^{*\prime}\end{bmatrix}' \\ &= \arg\min \begin{bmatrix}\alpha_{\mu_I}' & \alpha_{\mu_E}'\end{bmatrix}\left[\begin{bmatrix}\mu^I + \widetilde{\mu}^{I,A} \\ \mu^E + \widetilde{\mu}^{E,A}\end{bmatrix} + \begin{bmatrix}\widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}}\end{bmatrix}\begin{bmatrix}\alpha_{\mu_I} \\ \alpha_{\mu_E}\end{bmatrix}\right],\end{aligned}$$

subject to

$$\alpha_{\mu_i}' \geq 0,$$

$$\mu^i + \widetilde{\mu}^{i,A} + \widetilde{\mu}^{i,\varepsilon^{SP,j}}\alpha_{\mu_i} + \widetilde{\mu}^{i,\varepsilon^{SP,k}}\alpha_{\mu_k} \geq 0,$$

$i=\{I,E\}$. Here, μ^i and $\widetilde{\mu}^{i,A}$ denote, respectively, the steady-state value and the unrestricted relative impulse response of μ^i to a technology shock, that is, the impulse-response of μ^i when the collateral constraints are assumed to always bind. In this respect, the vector $\begin{bmatrix}\mu^I + \widetilde{\mu}^{I,A} \\ \mu^E + \widetilde{\mu}^{E,A}\end{bmatrix}$ contains the absolute, unrestricted impulse responses of the two shadow values stacked. Further, each matrix $\widetilde{\mu}^{i,\varepsilon^{SP,k}}$ contains the relative impulse responses of μ^i to shadow price shocks to agent k's constraint for $i,k=\{I,E\}$, in the sense that column s in $\widetilde{\mu}^{i,\varepsilon^{SP,k}}$ represents the response of the shadow value to a shock $\varepsilon_{j,t-j}^{SP,i}$, i.e. to a shadow price shock that hits in period j but is anticipated at time 0, as described above.[28] The off-diagonal elements of the matrix $\begin{bmatrix}\widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}}\end{bmatrix}$ take into account that the impatient household may be affected if the collateral constraint of the entrepreneur becomes non-binding, and *vice versa*. Following the discussion in Holden and Paetz (2012), a sufficient condition for the existence of a unique solution to the optimization problem is that the matrix $\begin{bmatrix}\widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}}\end{bmatrix} + \begin{bmatrix}\widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}}\end{bmatrix}'$ is positive definite. We have checked and verified that this condition is in fact always satisfied.

We can explain the nature of the optimization problem as follows. First, note that $\mu^i + \widetilde{\mu}^{i,A} + \widetilde{\mu}^{i,\varepsilon^{SP,i}}\alpha_{\mu_i} + \widetilde{\mu}^{i,\varepsilon^{SP,k}}\alpha_{\mu_k}$ denotes the combined response of μ_t^i to a given shock (here, a technology shock) *and* a simultaneous announcement of a set of future shadow price shocks for a given set of weights. Given the constraints of the problem, the objective is to find a set of optimal weights so that the impact of the (non-negative) shadow-price shocks is exactly large enough to make sure that the response of μ_t^i is never negative. The minimization ensures that the impact of the shadow price shocks will never be larger than necessary to obtain this. Finally, we only allow for solutions for which the value of the objective function is zero. This ensures that at any given horizon, positive shadow price shocks occur if and only if at least one of the two constrained variables, μ_t^I and μ_t^E, are at their lower bound of zero in that period. As pointed out by Holden and Paetz (2012), this can be thought of as a complementary slackness condition on the two inequality

[28] Each matrix $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$ needs to be a square matrix, so if the number of periods in which we guess the constraints may be non-binding is smaller than the number of periods for which we compute impulse responses, $T < T^{IRF}$, we use only the first T rows of the matrix, i.e., the upper square matrix.

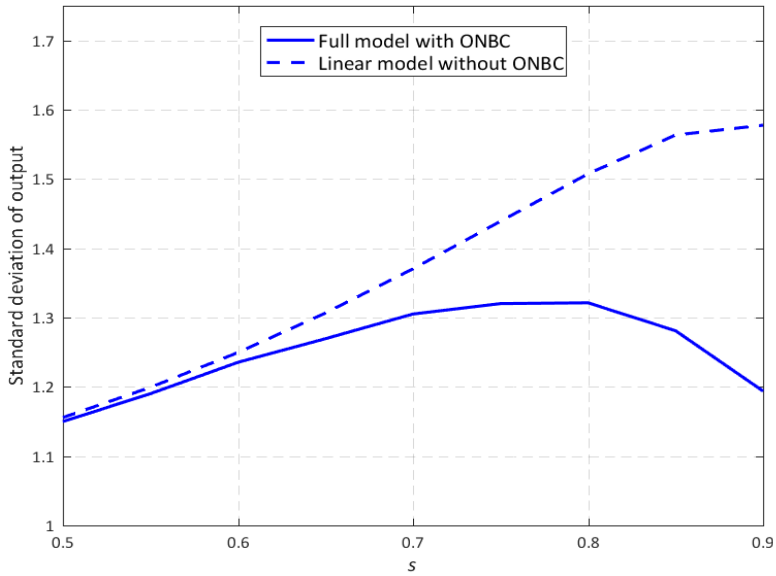

Figure D.1: *The importance of occasionally non-binding credit constraints.*
Notes: The figure illustrates the effect on output volatility of taking non-binding constraints into account. Numbers are median values from 501 stochastic model simulations of 2000 periods.

constraints of the optimization problem. Once we have solved the minimization problem, it is straightforward to compute the bounded impulse responses of all endogenous variables by simply adding the optimally weighted shadow price shocks to the unconstrained impulse responses of the model in each period.

We rely on the same method to compute dynamic simulations. In this case, however, we need to allow for more than one type of shock. For each period t, we first generate the shocks hitting the economy. We then compute the unrestricted path of the endogenous variables given those shocks and given the simulated values in $t-1$. The unrestricted paths of the bounded variables (μ_t^I and μ_t^E) then take the place of the impulse responses in the optimization problem. If the unrestricted paths of μ_t^I and μ_t^E never hit the bounds in future periods, our simulation for period t is fine. If the bounds are hit, we follow the method above and add anticipated shadow price shocks for a sufficient number of future periods. We then compute restricted values for all endogenous variables, and use these as our simulation for period t. Note that, unlike the case for impulse responses, in our dynamic simulations not all anticipated future shadow price shocks will eventually hit the economy, as other shocks may occur before the realization of the expected shadow price shocks and push the restricted variables away from their bounds.

As discussed in the main text, taking occasionally binding constraints into account is important also from a quantitative viewpoint, as entrepreneurs become unconstrained as much as 66% of the time for the highest LTV ratios we consider. To shed further light on this aspect, Figure D.1 displays output volatility as a function of the steady-state LTV ratio, as in Figure 2 in the main text, along with the corresponding statistics without accounting for occasionally binding constraints, i.e., from a counterfactual model in which the collateral constraints are simply treated as equalities. As the average LTV ratio is raised, the approximation error arising from treating the constraints as always binding increases substantially. In particular, output volatility increases monotonically with the average LTV ratio when credit constraints are treated as always binding. As described in the main text, raising the average LTV ratio involves a tradeoff between increasing the exposure of constrained agents to fluctuations in the value of their collateral and decreasing the frequency with which these agents are constrained. An approximation that treats collateral constraints as always binding misses the second leg of this tradeoff.

E Data and estimation

This appendix contains details about the data used for the estimation of the model, as well as the estimation procedure itself.

E.1 Data description

As described in the main text, we use data for the following five macroeconomic variables of the U.S. economy spanning the period 1980:Q1-2016Q2: Real GDP, real private consumption, real non-residential investment, real house prices, and the average of the two LTA series reported in Figure 1. All data series are taken from the Federal Reserve's FRED database. The series are the following:

- *Real Gross Domestic Product*, billions of chained 2009 dollars, seasonally adjusted, annual rate (series name: GDPC1).
- *Real Personal Consumption Expenditures*, billions of chained 2009 dollars, seasonally adjusted, annual rate (series name: PCECC96).
- *Real private fixed investment: Nonresidential* (chain-type quantity index), index 2009=100, seasonally adjusted (series name: B008RA3Q086SBEA).[29]
- *FHFA All Transactions House Price Index*, index 1980Q1=100, not seasonally adjusted (series name: USSTHPI).
  - To obtain the house price in real terms, this series is deflated using the GDP deflator (*Gross Domestic Product: Implicit Price Deflator*, index 2009=100, seasonally adjusted, series name: GDPDEF).
- LTA data: See Appendix A. We use the average of the two series displayed in Figure 1 from 1980:Q1 onwards.

We detrend all data series before estimation using a standard HP filter (with $\lambda = 1600$). In Figure E.1, we display all the detrended variables used in the estimation.

E.2 Estimation strategy

We use 13 empirical moments in the SMM estimation: The standard deviations and first-order autoregressive parameters of each of the five variables desribed above, and the correlation of consumption, investment, and house prices with output. These moments are matched to their simulated counterparts from the theoretical model. Our estimation procedure seeks to minimize the sum of squared deviations between empirical and simulated moments. As some of the moments are measured in different units (e.g., standard deviations and correlations), we use the percentage deviation from the empirical moment in each case. In order for the minimization procedure to converge, it is crucial to use the same set of shocks repeatedly, making sure that the only change in the simulated moments from one iteration to the next is that arising from updating the parameter values. In practice, since the list of parameter values to be estimated includes the variance of the shocks in the model, we draw from the standard normal distribution with zero mean and unit variance, and then scale the shocks by the variance of each of the three

[29] The most recent observations of this series, not yet available in the FRED database, have been collected directly from the Bureau of Economic Analysis, Table 5.3.3 in the NIPA tables.

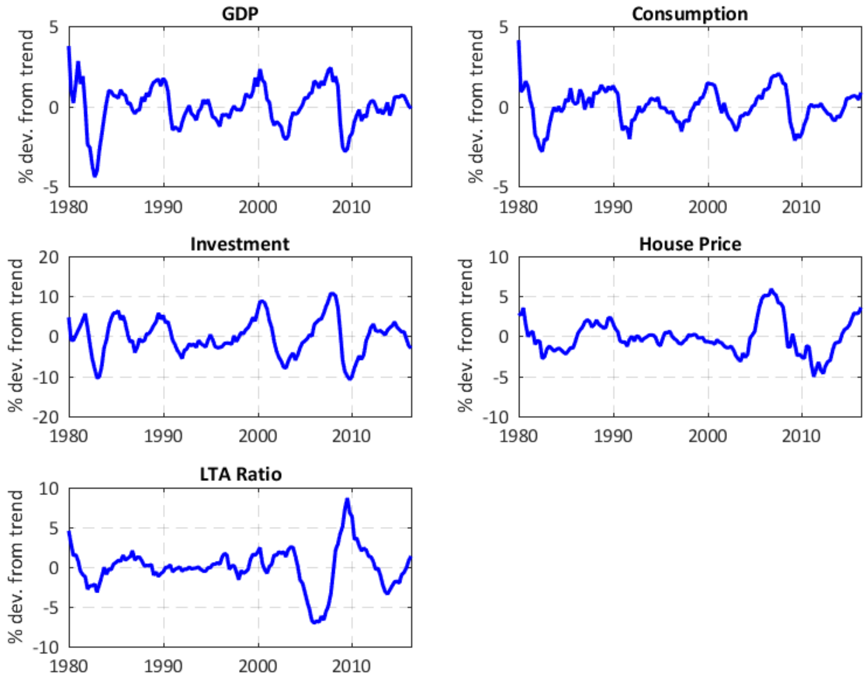

Figure E.1: *Data used in the estimation. The sample period is 1980Q1–2016Q2.*
Source: See text.

shock processes, allowing us to estimate the latter. We use a draw of 2000 realizations of each of the three shocks in the model, thus obtaining simulated moments for 2000 periods. To make sure that the draw of shocks used is representative of the underlying distribution, we make 501 draws of potential shock matrices, rank these in terms of the standard deviations of each of the three shocks, and select the shock matrix closest to the median in all cases. This matrix of shocks is then used in the estimation.

To initiate the estimation procedure a set of initial values for the estimated parameters are needed. These are chosen based on values reported in the existing literature. It is important to state that the estimation results proved robust to changes in the set of initial values, as long as these remain within the range of available estimates. Based on the empirical studies discussed in Section 3, we set the initial values of the investment adjustment cost parameter (Ω) and the habit formation in consumption (ρ) to 4 and 0.7, respectively. For the technology shock, we choose values similar to those used in most of the real business cycle literature, $\rho_A = 0.97$ and $\sigma_A = 0.005$ (see., e.g., Mandelman *et al.*, 2011). For the credit limit shock, we set the persistence parameter $\rho_s = 0.98$, while the standard deviation is set to $\sigma_s = 0.01$, consistent with the values estimated by Jermann and Quadrini (2012) and Liu *et al.* (2013). Finally, for the land-demand shock, we set $\rho_\varepsilon = 0.96$ and $\sigma_\varepsilon = 0.06$, in line with Iacoviello and Neri (2010) and Liu *et al.* (2013).

As discussed in the main text, we abstain from using an optimal weighting matrix in our estimation. Instead, we use the identity matrix, thus weighing all moment conditions equally. This is motivated by the findings of Altonji and Segal (1996), who show that when GMM is used to estimate covariance structures and, potentially, higher-order moments such as variances, the use of an optimal weighting matrix causes a severe downward bias in estimated parameter values. Similar concerns apply to SMM as to GMM. The bias arises because the moments used to fit the model itself are correlated with the weighting matrix, and may thus be avoided by the use of equally weighted minimization. Altonji and Segal (1996) demonstrate that equally weighted minimization schemes clearly dominate optimally weighted ones in such circumstances, and we therefore use equal weights, as we are interested in estimating, among other things, the covariance between macroeconomic variables. Ruge-Murcia (2012) points out that parameter estimates remain consistent when the identity matrix is used, and finds that the accuracy and efficiency gains associated with an optimal weighting matrix are not overwhelming.

When computing standard errors, we rely on a version of the delta method, as described, e.g., in Hamilton (1994). We approximate the numerical derivative of the moments with respect to the estimated parameters using the secant that can be computed by adding and subtracting ϵ to/from the estimates, where ϵ is a very small number. The covariance (or spectral density) matrix is estimated using the Newey-West estimator. As shown in Panel B of Table 1 in the main text, most parameters are fairly precisely estimated, with the exceptions of the investment adjustment cost parameter, Ω, and the standard deviation of the land preference shock, σ_{ε}. These two parameters are crucial for the model's ability to match the standard deviations of investment and the house price, respectively, but relatively unimportant for all other moments, penalizing them when the delta method is applied.

Table 2: Empirical and simulated moments

	Standard deviations (percent)	
	Model simulations	US data, 1980–2016
Output	1.30	1.31
Consumption	1.05	1.06
Investment	4.57	4.59
House price	2.26	2.16
LTV ratio	2.45	2.55
	Autocorrelations	
Output	0.78	0.84
Consumption	0.68	0.82
Investment	0.93	0.91
House price	0.67	0.92
LTV ratio	0.72	0.93
	Correlations with output	
Consumption	0.92	0.87
Investment	0.79	0.80
House price	0.52	0.51

F Additional figures

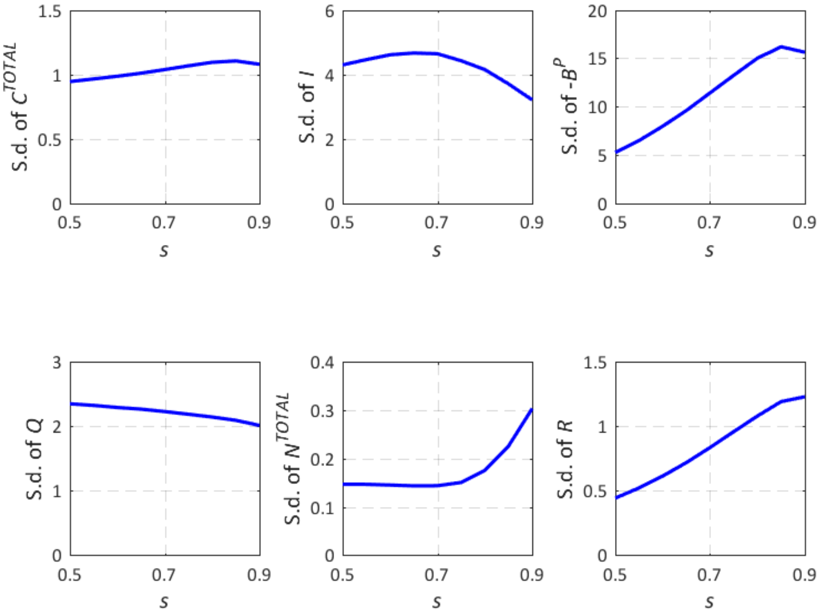

Figure F.1: *Standard deviations of main variables for different LTV ratios.*
Note: See the notes to Figure 2.

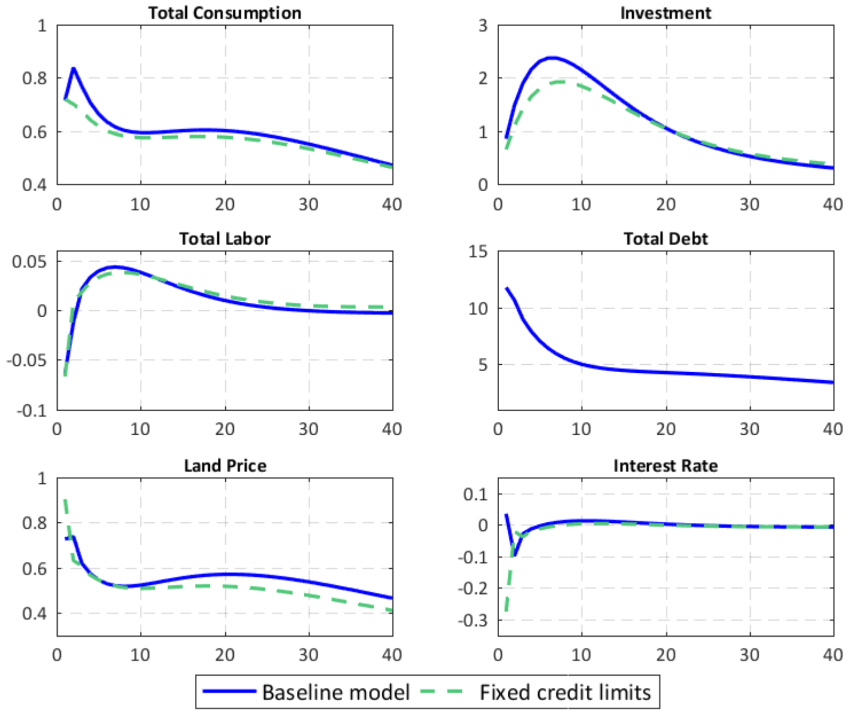

Figure F.2: Impulse responses of selected key variables to a positive one standard deviation technology shock.

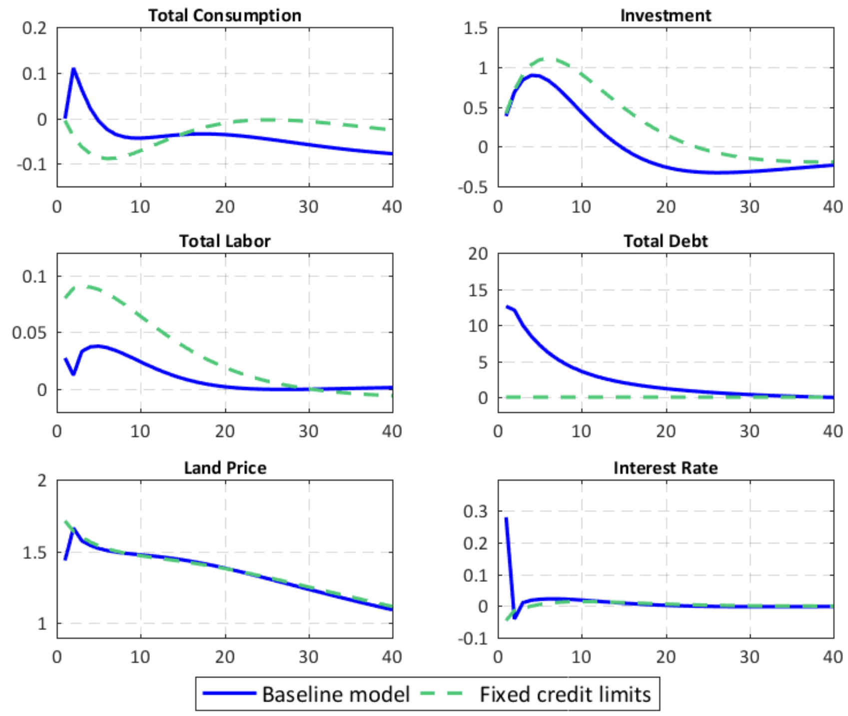

Figure F.3: Impulse responses of selected key variables to a positive one standard deviation land demand shock.

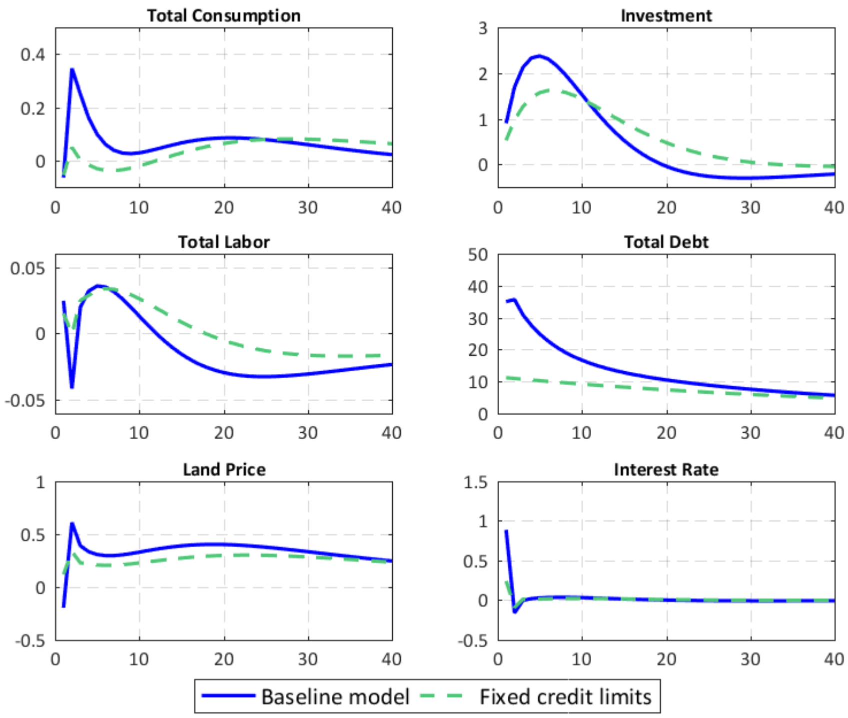

Figure F.4: Impulse responses of selected key variables to a positive one standard deviation credit limit shock.

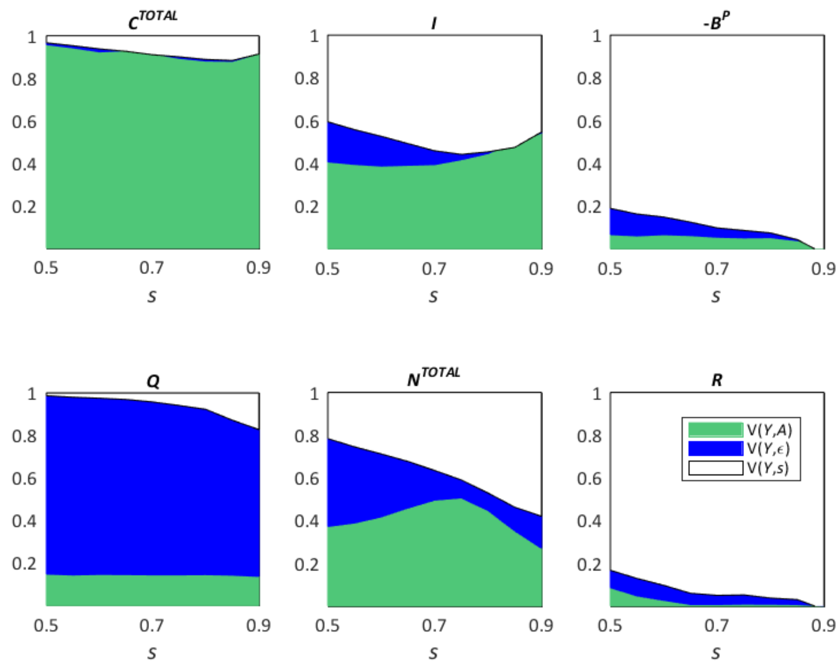

Figure F.5: *Variance contributions to fluctuations of selected key variables of each of the three shocks in the model for different LTV ratios.*
Note: See the notes to Figure 2.