Deepening Contractions and Collateral Constraints[*]

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Abstract

Since the mid-1980s we have assisted to a marked reduction in the volatility of business fluctuations in the US – a phenomenon usually labeled as the Great Moderation. This is not the only major modification in the shape of the business cycle: We document that—even excluding the Great Recession—the recessions taking place over the same period have been relatively more severe than before, reflecting into the skewness of the business cycle becoming increasingly negative. This finding can be explained by the concurrent increase in leverage of both households and firms. To demonstrate this point, we devise a DSGE model with collateralized borrowing and occasionally non-binding credit constraints. Easier credit access increases the likelihood that constraints become slack in the face of expansionary shocks, while contractionary shocks are further amplified due to tighter constraints. As a result, busts gradually become deeper than booms. Based on the differential impact that occasionally non-binding constraints exert on the shape of expansions and contractions, we are also able to reconcile a more negatively skewed business cycle with a moderation in its volatility. Finally, our model can account for an intrinsic feature of economic downturns preceded by private credit build-ups: Financially driven expansions lead to deeper contractions, as compared to equally-sized non-financial expansions.

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1 Introduction

Economic fluctuations across the industrialized world are typically characterized by asymmetries in the shape of expansions and contractions in aggregate activity. A prolific literature has extensively studied the statistical properties of this phenomenon, reporting that, relative to expansions, contractions are periods of larger and negative output fluctuations; see, among others, Neftci (1984), Hamilton (1989), Sichel (1993) and, more recently, Morley and Piger (2012). To account for this phenomenon, most of these studies consider static measures of skewness in economic aggregates, implicitly assuming that the shape of the business cycle does not change over time. This paper shows that US recessions have in fect become increasingly severe in the last three decades, thus reflecting into a more negatively skewed business cycle.

Explaining this pattern represents a challenge for existing business cycle models. To meet this, a theory is needed that involves non-linearities as well as a secular development capable of shaping the evolution in the skewness of the business cycle. In this respect, the importance of borrowing constraints as a source of business cycle asymmetries has long been recognized in the literature; see, e.g., the survey by Brunnermeier *et al.* (2013). In expansions, credit-constrained households and firms occasionally find themselves unconstrained, whereas credit constraints tighten during recessions. This non-linearity translates into a negatively skewed business cycle. In this respect, the past decades have witnessed a massive deregulation of financial markets, with one result being a substantial increase in the loan-to-value (LTV) ratios of both households and firms. This finding is confirmed by widespread evidence produced by Jordà *et al.* (2016), who report surging leverage in advanced economies in the last four decades, as well as positive correlation between the skewness of real GDP growth and the credit-to-GDP ratio. We propose additional evidence on this connection and suggest that the progressive increase in leverage is at the root of the increasingly negative skewness of the business cycle.

To account for these facts we design a DSGE model that allows for the possibility that the collateral constraints faced by firms and a fraction of the households do not always bind. Output skewness is practically zero going from low to intermediate values of the average LTV ratio, as collateral constraints tend to bind in either cyclical phase. Further increasing the average LTV ratio raises the likelihood of financial constraints becoming slack in the face of expansionary shocks, dampening the magnitude of the resulting boom. On the other hand, in the face of contractionary shocks, borrowers remain financially constrained, making their debt reduction increasingly burdensome. This, in turn, deepens contractions, so that the skewness of the business cycle becomes progressively negative, as in the reported evidence. We also show

that this pattern is transmitted onto aggregate consumption and investment dynamics.

Our findings carry important information about recent changes in the shape of business fluctuations. To elaborate on this, we juxtapose the drop in the skewness of the business cycle with the Great Moderation in macroeconomic volatility. While increasing LTV ratios cannot necessarily be pointed to as a main driver of the Great Moderation, our model reconciles the increase in the asymmetry of the business cycle with a drop in its volatility. In line with recent empirical evidence reported by Gadea-Rivas *et al.* (2014, 2015), neither changes to the depth nor to the frequency of recessionary episodes account for the stabilization of macroeconomic activity. In fact, the adjustment in macroeconomic volatility mostly rests on the characteristics of the expansions, whose magnitude declines as an effect of collateral constraints becoming increasingly non-binding in the face of higher credit limits.

Recently, increasing attention has been devoted to the connection between the driving factors behind business cycle expansions and the extent of the subsequent contractions. Jordà *et al.* (2013) report that more credit-intensive expansions tend to be followed by deeper recessions—irrespective of whether the latter are accompanied by a financial crisis. Our model accounts for this feature along two dimensions. First, we show that contractions become increasingly deeper as the average LTV ratio increases, even though the boom-bust cycle is generated by the same combination of expansionary and contractionary shocks. Second, we show that financially-driven expansions lead to deeper contractions, as compared with similar-sized expansions generated by non-financial shocks. Both exercises emphasize that, following a contractionary shock, the repercussion of constrained agents' deleveraging increases in the size of their debt. As a result, increasing leverage makes it harder for savers to compensate for the drop in consumption and investment of constrained agents. This narrative of the boom-bust cycle characterized by debt overhang is consistent with the results of Mian and Sufi (2010), who identify a close connection at the county level in the US between pre-crisis household leverage and the severity of the Great Recession.

The idea that occasionally binding credit constraints may give rise to macroeconomic asymmetries is not new, and has recently been examined in detail by Guerrieri and Iacoviello (2014), Jensen *et al.* (2016), and Maffezzoli and Monacelli (2015).[1] Compared to Guerrieri and Iacoviello (2014), whose focus is on the recent boom-bust cycle in the US housing market and its connection with private consumption, we examine the impact of secular variations in both

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[1]This idea is also closely related to the 'sudden stop' literature, in which a small open economy faces an occasionally binding constraint on its access to external credit. See, e.g., Mendoza (2010) and Benigno *et al.* (2013).

households' and firms' credit limits on the shape of output fluctuations. In this respect, our study implies that non-binding credit constraints are likely to have become a more salient feature of the macroeconomy in recent decades. This intuition is indirectly supported by Guerrieri and Iacoviello (2014), who show that non-binding credit constraints were prevalent during the last pre-crisis boom in the US. Maffezzoli and Monacelli (2015) provide an extensive account of the characteristics of financially-driven contractions, and also report that the aggregate implications of deleverage shocks are state-dependent, with the economy's response being greatly amplified in situations where agents switch from being financially unconstrained to being constrained. However, while Maffezzoli and Monacelli (2015) focus on the characteristics of drops in economic activity induced by financial shocks and conditional on different degrees of firm leverage, we design our experiments so as to generate boom-bust cycles where expansions can be either credit-fueled or driven by non-financial shocks. In line with the boom-bust episodes studied by Jordà et al. (2013), we show that the nature of the driving forces behind a given expansion are crucial for predicting the deepness of the ensuing contraction.

Regarding the connection between financial liberalization and business cycle asymmetry, our paper is related to a recent empirical literature. Popov (2014) studies business cycle asymmetry in a large panel of developed and developing countries. Two main results are documented. First, the average business cycle skewness across all countries became markedly negative after 1991, consistent with our findings for the US. Second, this pattern is particularly distinct in countries that liberalized their financial markets. Bekaert and Popov (2015) examine a large cross-section of countries, reporting that more financially developed economies have more negatively skewed business cycles. Ordoñez (2013) documents that countries with more developed financial markets display less asymmetry than countries with a more rudimentary financial system. While this finding is at odds with our results and those of the aforementioned papers, Ordoñez (2013) does not focus on the business cycle effects of a secular process of financial development in industrialized countries. Moreover, he considers financial development as improved monitoring, which alleviates amplification of negative shocks, whereas we consider increasing credit limits, resulting in collateral constraints becoming non-binding more often. Rancière et al. (2008) establish a cross-country link between real GDP growth and the skewness of credit growth—a link which is stronger in financially liberalized countries. While we focus on the asymmetry of output, our credit measure shares this property, making our results comparable with their findings.

The rest of the paper is organized as follows. We present empirical evidence of changes in

business cycle skewness and loan-to-value ratios in Section 2. Section 3 presents our model. Section 4 discusses our main result and connects our findings to the Great Moderation in economic volatility. Section 5 shows that the model is capable of producing the type of debt overhang recession emphasized in recent empirical studies. Section 6 concludes. The Appendices contain supplementary material concerning the model solution and empirical details.

2   Empirical evidence

This section presents a set of stylized facts that motivate the analysis. We first present evidence on the increasing violence of recessionary episodes in the post-Great Moderation period, coupling it with the observation that the skewness of the growth of real GDP has become increasingly negative over the last three decades. We next show how the concurrent deregulation of financial markets has reflected into a substantial increase in the loan-to-value (LTV) ratios of both households and firms. Finally, we take advantage of cross-sectional variation across US States to document a solid empirical relationship between household leverage and the deepness of State-level contractions during the Great Recession.

2.1   Business cycle evidence

Following the seminal work of Mitchell (1927), McKay and Reis (2008) measure 'violence' as the average fall of real GDP during a recessionary episode. The first column of Table 1 reports this metric for the recessions after 1960.

In line with the available anecdotal evidence, it is confirmed that the 1991 and 2001 recessions have been rather mild in terms of violence, as compared with both the Great Recession and most of the pre-1984 contractions. However, since the seminal paper of McConnell and Perez-Quiros (2000) it has been widely documented that the US business cycle has become less volatile starting from the mid-1980s. In light of this, it seems appropriate to control for the volatility of the business cycle when comparing recessionary episodes at different points in time. Thus, the second column of Table 1 also reports a measure of 'standardized violence', obtained by weighing violence for the variability of year-on-year GDP growth in the 5 years prior to the recession.[2] Under this metric we get a rather different picture. The three recessionary episodes occurred during the Great Moderation period are substantially more violent than the pre-1984

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[2]The variability is calculated as the standard deviation of the year-on-year growth rate of real GDP over a 5-year period. We also exclude the period running up to the recession by calculating the standard deviation up to a year before the recession starts. Calculating the standard deviation on quarterly growth rates of real GDP returns a qualitatively similar picture.

Table 1. The violence of US recessions (1960-2009).		
	Violence	Std. Violence
1960 (II) – 1961 (I)	1.8009	0.5718
1969 (IV) – 1970 (IV)	0.4710	0.2674
1973 (IV) – 1975 (I)	2.5293	1.2275
1980 (I) – 1980 (III)	4.4006	1.7747
1981 (III) – 1982 (IV)	1.5130	0.6365
1990 (III) – 1991 (I)	2.6511	3.6238
2001 (I) – 2001 (IV)	1.2671	1.3613
2007 (IV) – 2009 (II)	2.8909	3.2966

Notes: For every recession we calculate 'Violence' as the annualized fall of real GDP from the peak to the trough of the contractionary episode, dividing by the length of the recession. 'Std. Violence' standardizes the violence of the recession by the average business cycle volatility prior to the recession itself. The latter is calculated as the standard deviation of the year-on-year real GDP growth in the 5 years prior to each recession (the last 4 quaters are discarded).

Table 1: The violence of US recessions (1960-2009)

ones:[3] averaging out over the first five recessionary episodes returns an annualized violence of 1.44%, against an average of 2.76% for the post-1984 period (2.49%, if we exclude the Great Recession).

This evidence implies that the post-1984 period is characterized by less volatile, yet increasingly negatively skewed business cycles. To elaborate further on this, we calculate the skewness of real GDP growth in the pre- and post-1984 period, as well as the corresponding volatility. These are displayed in Figure 1. As for volatility, we observe a substantial drop in the post-1984 sample, in line with the Great Moderation narrative. Notably, the estimated skewness is not significantly different from zero in the first part of the sample, while displaying a significantly negative coefficient in the more recent past. To corroborate this finding we also compute a measure of time-varying skewness of real GDP growth using a nonparametric estimator, in the spirit of Giraitis *et al.* (2014).[4] Figure 1 also shows both the time-varying volatility and skewness of GDP growth. Notably, the second half of the sample is associated with more negative skewness, thus reflecting a more pronounced severity of economic contractions. This finding is broadly confirmed when computing the time-varying skewness of other key macroeconomic aggregates (see Figure 2).

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[3] Due to Volcker's disinflationary policy, the two recessions at the beginning of the 1980s are often regarded as a single recessionary episode. The standardized violence for the two recessions analyzed as one is 0.0532.

[4] This approach simply consists of calculating volatility and skewness of the data in a recursive fashion, reweighting the data using a Gaussian kernel, which emphasizes the local dynamics of the series. See Appendix A for a more detailed discussion.

Figure 1. Time-varying volatility and the skewness of GDP growth.
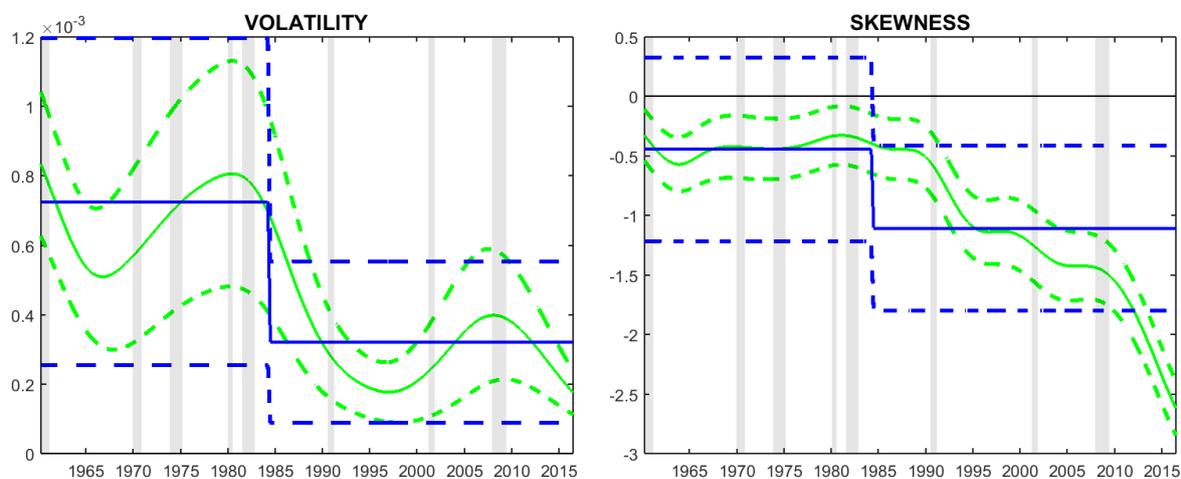
Notes. The green (light) line in Figure 1 reports the time-varying volatility and skewness of real GDP growth, obtained by using a nonparametric estimator in the spirit of Giraitis *et al.* (2014), as well as the associated 68% confidence interval. The blue (dark) line in the figure denotes the point estimates of volatility and skewness in the pre- and post-1984 sample, as well as the associated 68% confidence interval. The vertical shadowed bands denote the NBER recession episodes.

Figure 2. Time-varying skewness of various macroeconomic aggregates.
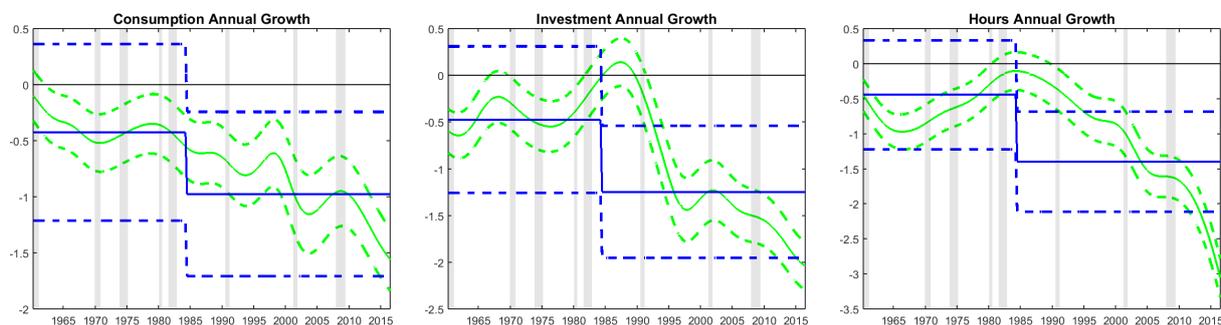

Notes. See notes to Figure 1.

2.2 Long-run trends in the LTV ratios

The past decades have witnessed a massive deregulation of financial markets, with one result being a substantial increase in leverage in advanced economies, as widely discussed by Jordà et al. (2016), among others. Figure 3 plots the loan-to-value (LTV) ratios of both households and the corporate sector in the US.[5]

Figure 3. The ratio of liabilities-to-assets for households and firms in the US.
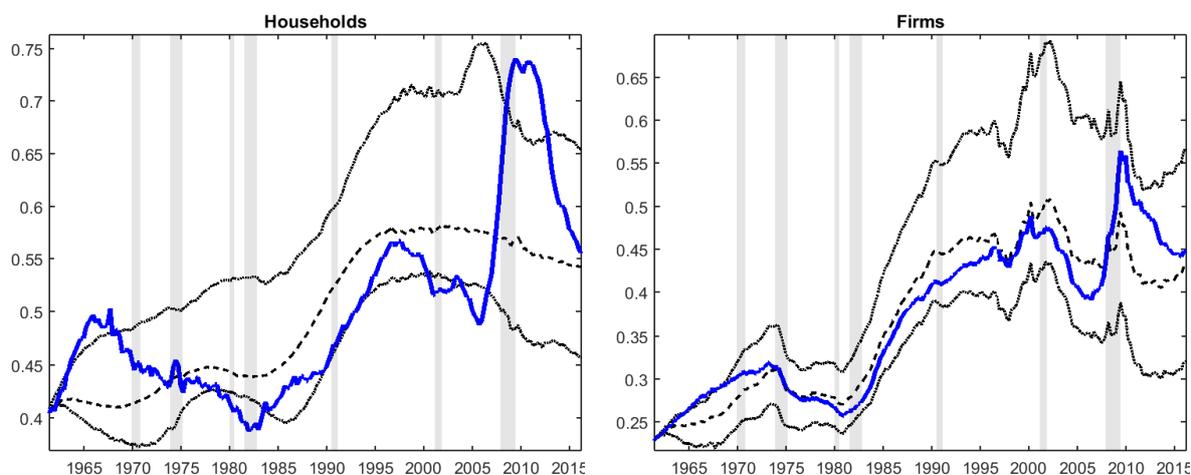
Notes. The continuous line graphs the ratio of liabilities-to-assets for households and firms, obtained from Flow of Funds data of the United States. The dashed line denotes the underlying long-run trend of the original series. The dotted lines represent the 68% confidence bands, calculated with 1000 bootstrap replications. The vertical shadowed bands denote the NBER recession episodes.

The two series are clearly nonstationary. More importantly, they are cointegrated, i.e. the secular movements are driven by a common long-run component.[6] Each panel of Figure 3 reports the trend obtained by means of a permanent-transitory decomposition, as indicated by Lettau and Ludvigson (2014). The intuition underlying this methodology is that a single one shock exerts a permanent effect on the level of a given variable. Thus, we generate the long-

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[5] As we discuss in Appendix B, the aggregate ratios of loans to assets reported in Figure 3 are likely to understate the actual LTV requirements faced by the marginal borrower. While alternative measures may yield higher *levels* of LTV ratios, they give rise to the same conclusions about the development of leverage over time (see also Graham *et al.*, 2014, and Jordà *et al.*, 2016).

[6] Using ADF tests we cannot reject the null that the two series are I(1), whereas we clearly reject the stationary hypothesis using the KPSS test. According to Johansen's trace test the two series are cointegrated at the 10% significance level. We also calculate the *p-value* of the test using sieve bootstrap techniques (see e.g. Buhlmann, 1997, and Chang *et al.*, 2005), confirming the results obtained through Johansen's procedure.

run counterpart of the LTV series by simulating the Vector Error Correction Model (VECM) subject only to the permanent shock. The corresponding 68% confidence interval is then calculated from 1000 bootstrap replications. Notably, both trends display a remarkable jump in the 1980s, rising from 40% to 55% in the household sector and from 25% to 45% in the corporate sector.[7,8]

2.3   Cross-State evidence

In the previous subsections we have established that the post-1984 period is associated with an increase in the severity of the contractionary episodes. Moreover, over the same period the process of financial deregulation has been associated with a sizeable increase in leverage of both households and firms. Relying on county-level US data, Mian and Sufi (2010) have identified a strong causal link between pre-crisis household leverage and the severity of the Great Recession. We now produce related evidence based on State-level data. Specifically, we take data on quarterly real GDP from the BEA Regional Economic Accounts and compute both the skewness of State-level GDP growth and the violence of the Great Recession in all the US States. To account for the fact that the recession has not started/ended in the same period throughout the US, we calculate the start of the recession as the period with the highest level of real GDP in the window that goes from 5 quarters before the NBER peak date to one quarter after that. Similarly, the end of the recession for a given state is calculated as the period with the lowest level of real GDP in the window from one quarter before to 5 quarters after the NBER trough date. Figure 4 correlates the resulting statistics to the average debt-to-income ratio prior to the recession.[9] Notably, the states where households were more deeply leveraged not only witnessed more severe GDP contractions during the last recession, but have

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[7]The corresponding cyclical components are highly correlated, as displayed Figure F.1 in Appendix F.

[8]We have also computed the long-run component of the LTV series following the procedure of Gonzalo and Granger (1995), as well as the Beveridge and Nelson (1981) decomposition. The latter is obtained by assuming a single cointegrating relationship between the two series and calculating their implied long-run forecast at each point in time (for further details, see Garratt *et al.*, 2006). These different methodologies yield analogous results. See Figure F.2 in Appendix F for further evidence.

[9]This variable has been constructed through the State Level Household Debt Statistics produced by the New York Fed.

also displayed a more negatively skewed GDP growth.


Figure 4. Leverage and asymmetry.
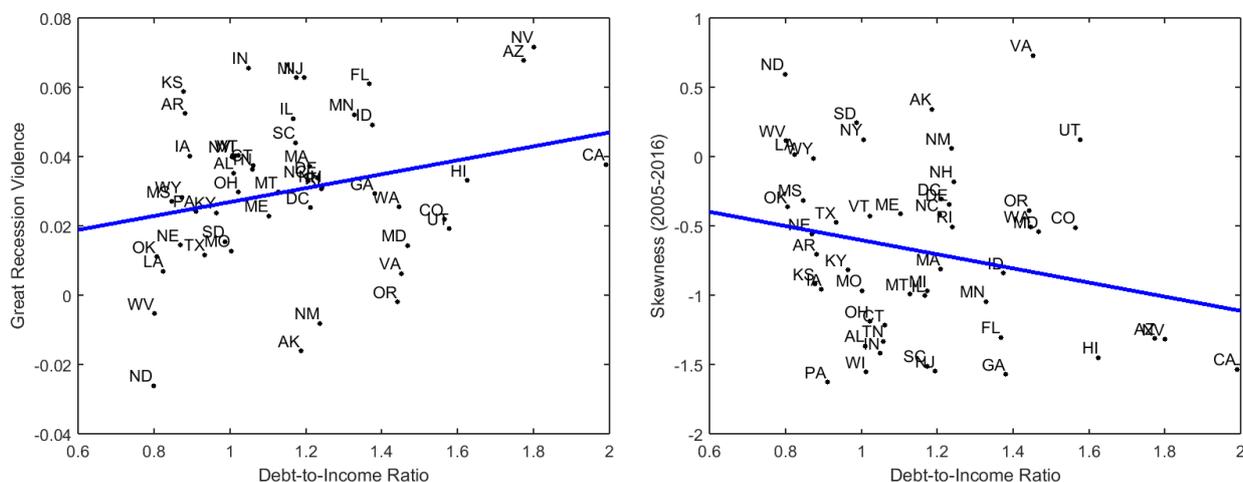
Notes. The left-hand panel plots the violence of the Great Recession in each US State against the average debt-to-income ratio at the household-level over the period 2003-07. In order to allow for the fact that the recession does not start/end at the same time throughout the US, we calculate the start of the recession as the period with the highest level of real GDP in the window that goes from 5 quarters before the NBER peak date to one quarter after that. Similarly the state-specific trough is calculated as the period with the lowest level achieved by the state-specific real GDP in the window from one quarter before to 5 quarters after the NBER trough date. The right-hand panel plots the skewness of real GDP growth over the 2003-2015 period against the average debt-to-income ratio.

To get further insight into the cross-sectional connection between the magnitude of the Great Recession and business cycle dynamics, we next order states according to the average debt-to-income ratio in the household sector.[10] We then construct two synthetic GDP series, computed as the median real GDP of the top and the bottom ten leveraged states, respectively, which we report in Figure 5. Clearly, whereas there are no noticeable differences in the performance of the two groups before and after the Great Recession, the subsequent drop in real activity has been much higher for more leveraged states.[11]

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[10] See Figure F.3 in Appendix F.

[11] We would get the same qualitative picture if we were to look at the top/bottom 15 states.

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Figure 5. GDP dynamics for highly and poorly leveraged US states.
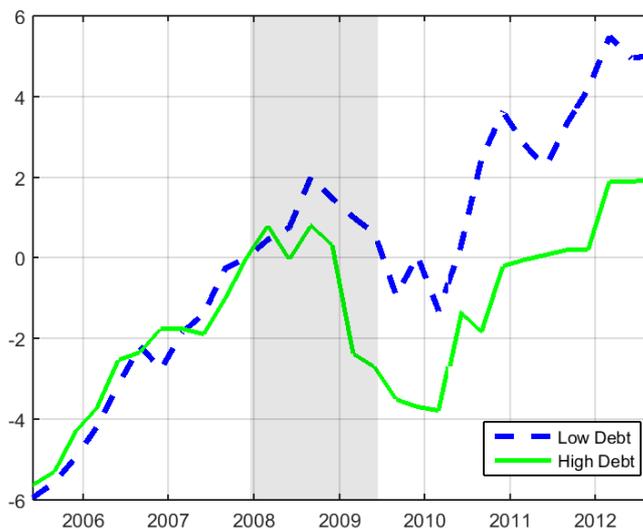
Notes. Figure 5 reports two synthetic GDP series obtained by ranking US states according to their average debt-to-income ratio in the 5 years before the Great Recession. The dashed line is obtained as the median real GDP of the top 10 states, while the continuous line is obtained as the median for the bottom 10 states. The resulting statistics have been normalized to zero at the beginning of the Great Recession (i.e.,2007-IV). The vertical shadowed band denotes the 2007(IV)-2009(II) recession episode.

3 The model

We now devise a structural model that can explain the empirical evidence described in the previous section. We adopt a standard real business cycle model augmented with collateral constraints along the lines of Kiyotaki and Moore (1997), Iacoviello (2005), Liu *et al.* (2013), and Justiniano *et al.* (2015), *inter alia*.[12] The economy is populated by three types of agents, each of mass equal to one. These agents differ by their discount factors, with the so-called patient households displaying the highest degree of time preference, while impatient households and entrepreneurs have relatively lower discount factors. As a result, patient households will be acting as lenders. Moreover, patient and impatient households supply labor, consume non-durable goods and land. Entrepreneurs only consume non-durable goods, and accumulate both land and physical capital, which they rent to firms. These are of unit mass and operate under perfect competition, taking labor inputs from both types of households, along with capital and

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[12] Jensen *et al.* (2016) employ this framework to examine the impact of secular changes in credit limits on business cycle volatility and comovement between private debt and consumption/investment dynamics.

land from the entrepreneurs. The resulting gross product may be used for investment and non-durable consumption.

3.1 Patient households

The utility function of patient households is given by:

$$\mathrm{E}_0\left\{\sum_{t=0}^{\infty}\left(\beta^P\right)^t\left[\log\left(C_t^P-\rho^P C_{t-1}^P\right)+\varepsilon_t\log\left(H_t^P\right)+\frac{\nu^P}{1-\varphi^P}\left(1-N_t^P\right)^{1-\varphi^P}\right]\right\},\quad \varphi^P\neq 1,\quad (1)$$

where C_t^P denotes non-durable consumption, H_t^P is the stock of land, and N_t^P denotes the fraction of time devoted to labor. Moreover, $0<\beta^P<1$ is the discount factor and $\varphi^P>0$ are the coefficients of relative risk aversion pertaining to non-durable consumption, land services and leisure, respectively, and $\nu^P>0$ is the weight of labor disutility. Finally, ε_t is a land-preference shock satisfying

$$\log\varepsilon_t=\log\varepsilon+\rho_\varepsilon\left(\log\varepsilon_{t-1}-\log\varepsilon\right)+u_t,\qquad 0<\rho_\varepsilon<1,\qquad (2)$$

where $\varepsilon>0$ denotes the steady-state value and where $u_t\sim\mathcal{N}\left(0,\sigma_\varepsilon^2\right)$. Utility maximization is subject to the following budget constraint

$$C_t^P+Q_t\left(H_t^P-H_{t-1}^P\right)+R_{t-1}B_{t-1}^P=B_t^P+W_t^P N_t^P,\qquad (3)$$

where B_t^P denotes the stock of one-period debt held at the end of period t, R_t is the gross real interest rate on debt, Q_t is the price of land in units of consumption goods, and W_t^P is the real wage.

3.2 Impatient households

The utility of impatient households takes the same form as that of patient households:

$$\mathrm{E}_0\left\{\sum_{t=0}^{\infty}\left(\beta^I\right)^t\left[\log\left(C_t^I-\rho^I C_t^I\right)+\varepsilon_t\log\left(H_t^I\right)+\frac{\nu^I}{1-\varphi^I}\left(1-N_t^I\right)^{1-\varphi^I}\right]\right\},\quad \varphi^I>0,\ \varphi^I\neq 1,\ \nu^I>0,$$

$$(4)$$

where, as for the patient households, C_t^I denotes non-durable consumption, H_t^I is the stock of land, and N_t^I denotes the fraction of time devoted to labor. Households' different impatience is captured by assuming $\beta^P>\beta^I$. This ensures that, in the steady state, patient and impatient

households act as lenders and borrowers, respectively. Impatient households are also subject to the following budget constraint

$$C_t^I + Q_t \left(H_t^I - H_{t-1}^I \right) + R_{t-1} B_{t-1}^I = B_t^I + W_t^I N_t^I. \tag{5}$$

Moreover, impatient households are subject to a collateral constraint, according to which their borrowing B_t^I is bounded above by a fraction s_t of the expected present value of durable goods holdings at the beginning of period $t + 1$:

$$B_t^I \leq s_t \frac{\mathrm{E}_t \left\{ Q_{t+1} \right\} H_t^I}{R_t}, \tag{6}$$

This constraint can be rationalized in terms of limited enforcement, as in Kiyotaki and Moore (1997) and Iacoviello (2005). The loan-to-value (LTV) ratio (or credit limit), s_t, is stochastic and aims at capturing financial shocks (see, e.g., Jermann and Quadrini, 2012 and Liu *et al.*, 2013):

$$\log s_t = \log s + \rho_s \left(\log s_{t-1} - \log s \right) + v_t, \qquad 0 < \rho_s < 1, \tag{7}$$

where $v_t \sim \mathcal{N} \left(0, \sigma_s^2 \right)$ and s, the steady-state LTV ratio, is a proxy for the *average* stance of credit availability.

3.3 Entrepreneurs

Entrepreneurs have preferences over non durables only (cf. Iacoviello, 2005; Liu *et al.*, 2013), and maximize

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\beta^E \right)^t \log \left(C_t^E - \rho^E C_{t-1}^E \right) \right\}, \tag{8}$$

where C_t^E denotes entrepreneurial non-durable consumption and $\beta^P > \beta^E$. Utility maximization is subject to the following budget constraint

$$C_t^E + I_t + Q_t \left(H_t^E - H_{t-1}^E \right) + R_{t-1} B_{t-1}^E = B_t^E + r_{t-1}^K K_{t-1} + r_{t-1}^H H_{t-1}^E, \tag{9}$$

where I_t denotes investment in physical capital, K_{t-1} is the physical capital stock rented to firms at the end of period $t - 1$, and H_{t-1}^E is the stock of land rented to firms. Finally, r_{t-1}^K and r_{t-1}^H are the rental rates on capital and land, respectively. Capital accumulation is given

by the law of motion

$$K_t = (1 - \delta) K_{t-1} + \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t, \qquad 1 > \delta > 0, \quad \Omega > 0, \tag{10}$$

whereby quadratic investment adjustment costs are assumed. Like impatient households, entrepreneurs are credit constrained, but they are able to use both capital and their holdings of land as collateral:[13]

$$B_t^E \le s_t \mathrm{E}_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\}, \tag{11}$$

where Q_t^K denotes the price of installed capital in consumption units. For simplicity, we assume that households and entrepreneurs are subject to common credit limits.[14]

3.4 Firms

Firms operate under perfect competition, employing a constant-returns-to-scale technology. They rent capital and land from the entrepreneurs and hire labor from both types of households in order to maximize their profits. The production technology for output, Y_t, is given by:[15]

$$Y_t = A_t \left[\left(N_t^P \right)^\alpha \left(N_t^I \right)^{1-\alpha} \right]^\gamma \left[\left(H_{t-1}^E \right)^\phi K_{t-1}^{1-\phi} \right]^{1-\gamma}, \qquad 0 < \alpha, \ \phi, \ \gamma < 1, \tag{12}$$

with total factor productivity A_t evolving according to

$$\log A_t = \log A + \rho_A \left(\log A_{t-1} - \log A \right) + z_t, \qquad 0 < \rho_A < 1, \tag{13}$$

where $A > 0$ is the steady-state value of A_t, and $z_t \sim \mathcal{N} \left(0, \sigma_A^2 \right)$.

3.5 Market clearing

Aggregate supply of land is fixed at H, implying that land-market clearing is given by

$$H = H_t^P + H_t^I + H_t^E. \tag{14}$$

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[13] The importance of real estate as collateral for business loans has recently been emphasized by Chaney *et al.* (2012) and Liu *et al.* (2013).

[14] In the model of Iacoviello (2005), the LTV ratio faced by entrepreneurs (0.89) is much higher than that faced by impatient households (0.55), while the opposite is the case in Gerali *et al.* (2010), who set 0.35 for entrepreneurs and 0.7 for households. As a crude approximation, we first assume that LTV ratios faced by households and firms are equal. In ongoing work, we break down this assumption.

[15] The assumption of imperfect substitutability between labor types follows Iacoviello (2005) and Justiniano *et al.* (2015), among others. Iacoviello and Neri (2010) note that perfect substitutability complicates the solution of their model substantially, but yields similar results.

The economy-wide net financial position is zero, such that

$$B_t^P + B_t^I + B_t^E = 0. \tag{15}$$

Finally, the aggregate resource constraint can be written as

$$Y_t = C_t^P + C_t^I + C_t^E + I_t. \tag{16}$$

3.6 Equilibrium and solution method

An equilibrium is defined as a sequence of prices and quantities which, conditional on the sequence of shocks $\{A_t, \varepsilon_t, s_t\}_{t=0}^{\infty}$ and the initial conditions, satisfy the optimality conditions, the budget and credit constraints, as well as the technological constraints and the market-clearing conditions. We provide the optimality conditions in Appendix C, while the steady state and the log-linearized version of the model are presented in the Online Appendix to this paper. Due to the assumptions about the discount factors, $\beta^P < \beta^I$ and $\beta^P < \beta^E$, both collateral constraints are binding in steady state. The steady-state real interest rate is pinned down by patient households' Euler equation, whereas impatient households and entrepreneurs have a higher subjective real rate of interest. However, the optimal level of debt of one or both agents may fall short of the credit limit when the model is not at its steady state, in which case the collateral constraint will be non-binding.

To account for the occasionally non-binding nature of the collateral constraints, our solution method follows Laséen and Svensson (2011) and Holden and Paetz (2012), who develop a solution method for log-linearized DSGE models featuring inequalities. The idea is to introduce a set of (anticipated) "shadow value shocks" to ensure that the shadow values associated with each of the two collateral constraints remain non-negative at all times. For first-order perturbations, our solution method produces similar simulated moments as the method of Guerrieri and Iacoviello (2014, 2015); cf. Holden and Paetz (2012). We present the technical details of the method in Appendix D.

3.7 Parameterization

We calibrate the model to match the quantitative characteristics of the U.S. business cycle. The calibrated parameters are summarized in Table 2, while Appendix E contains an extensive

Parameter	Interpretation	Value
	Preference parameters	
β^P	Discount factor, patient households	0.99
$\beta^i, i = \{I, E\}$	Discount factor, impatient agents	0.97
$\sigma_C^i, i = \{P, I, E\}$	CRRA coefficient for consumption	1
$\sigma_H^i, i = \{P, I\}$	CRRA coefficient for housing	1
$\sigma_H^i, i = \{P, I\}$	CRRA coefficient for labor	9
ε	Weight on housing utility	0.085
$\nu^i, i = \{P, I\}$	Weight on labor disutility	0.27
	Technology parameters	
γ	Labor share of production	0.7
α	Income share of patient households	0.7
ϕ	Non-labor input share of land	0.15
Ω	Investment adjustment cost parameter	4
δ	Capital depreciation rate	0.035
$n^i, i = \{I, E\}$	Mass of each type of agent	1/3
	Shock parameters	
ρ_A	Persistence of technology shock	0.97
ρ_s	Persistence of credit-limit shock	0.98
ρ_ε	Persistence of land-demand shock	0.96
σ_A	Std. dev. of technology shock	0.005
σ_s	Std. dev. of credit-limit shock	0.0119
σ_ε	Std. dev. of land-demand shock	0.06

Table 2: Calibrated parameter values

description of the parameter values.[16] As our perspective is on the secular behavior of business cycle asymmetry, the calibration strategy is designed so as to match a set of "big ratios" for the US economy as reported, e.g., by Liu *et al.* (2013): this implies a steady-state ratio of residential land to output around 1.45, a ratio of commercial land to output of 0.65 and a capital to output ratio of 1.15, all at the annual level. These values are compatible with $s = 0.7$.

To account for a gradual relaxation of both households' and firms' credit limits, we let the steady-state LTV ratio faced by households and entrepreneurs, s, vary over the range [0.3,0.9] and report statistics for 13 different values within this range.[17] In this way, we obtain a comprehensive picture of the effects of different LTV ratios on the macroeconomy. When we report impulse responses, however, we do so only for two values of s. The first is a "high" LTV

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[16]Our parameterization is close to the values estimated in Jensen *et al.* (2016), and is broadly in line with parameter values used in existing work (e.g., Iacoviello, 2005; Liu *et al.*, 2013). In ongoing work, we seek to estimate the model using Simulated Method of Moments.

[17]Changing s within this range allows us to match values of the big ratios that are close to the proposed calibration. For values of s below 0.3, the credit constraints become non-binding only very rarely, so that our statistic of main interest, the skewness of output, is always very close to zero. We have chosen the upper bound of the range to 0.9 so that given the process for s_t, actual LTV ratios remain below 1 in 95 percent of all periods. While LTV ratios above 1 may sometimes occur empirically, it is hard to reconcile with the idea of limited contract enforcement which we follow in this paper.

regime, where $s = 0.7$. The second, a "low" LTV regime, has $s = 0.35$.[18]

4 Asymmetric business cycles and collateral constraints

We are now ready to explore the ability of our model in generating stronger business cycle asymmetry as s increases. We do so in three steps. First, we inspect a set of impulse responses to build intuition around the non-linear transmission of different shocks. Next, we present the skewness of output and other variables implied by the model, based on a large number of stochastic simulations. Finally, we examine the behavior of skewness in conjunction with the Great Moderation in macroeconomic volatility.

4.1 Inspecting the mechanism: impulse responses

To gain a preliminary insight into the nature of the asymmetry generated by our framework, and how this evolves under different LTVs, we study the propagation of 'large' shocks, which have the potential to make the borrowing constraints non-binding.[19] Figure 6 displays the response of output to a set of large, positive shocks, as well as the mirror image of the response to equally-sized negative shocks, under a high and a low LTV ratio. Under a high average LTV ratio, a positive technology shock renders the borrowing constraint of the entrepreneurs slack for six quarters, while impatient households remain constrained throughout. Therefore, entrepreneurs optimally choose to borrow less than they are able to: This attenuates the expansionary effect on their demand for land and, in turn, dampens the boom in aggregate economic activity. On the contrary, following a negative technology shock, borrowing constraints remain binding throughout. As a result, impatient households and entrepreneurs are forced to cut back on their borrowing in response to the drop in the value of their collateral assets. This produces a stronger output response. In other words, under high LTV ratios a large, negative technology shock has a larger impact on output than a similar-sized positive shock when occasionally non-binding constraints are taken into account.

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[18]Both of these values are within the range of values typically used in the literature; e.g., Mendoza (2010) reports 0.2–0.3, Calza *et al.* (2013) use 0.6, Liu *et al.* (2013) report 0.75, while Justiniano *et al.* (2014) set a value of 0.8.

[19]In our stochastic simulations, instead, combinations of positive 'normal' shocks will be sufficient to make the constraints non-binding.
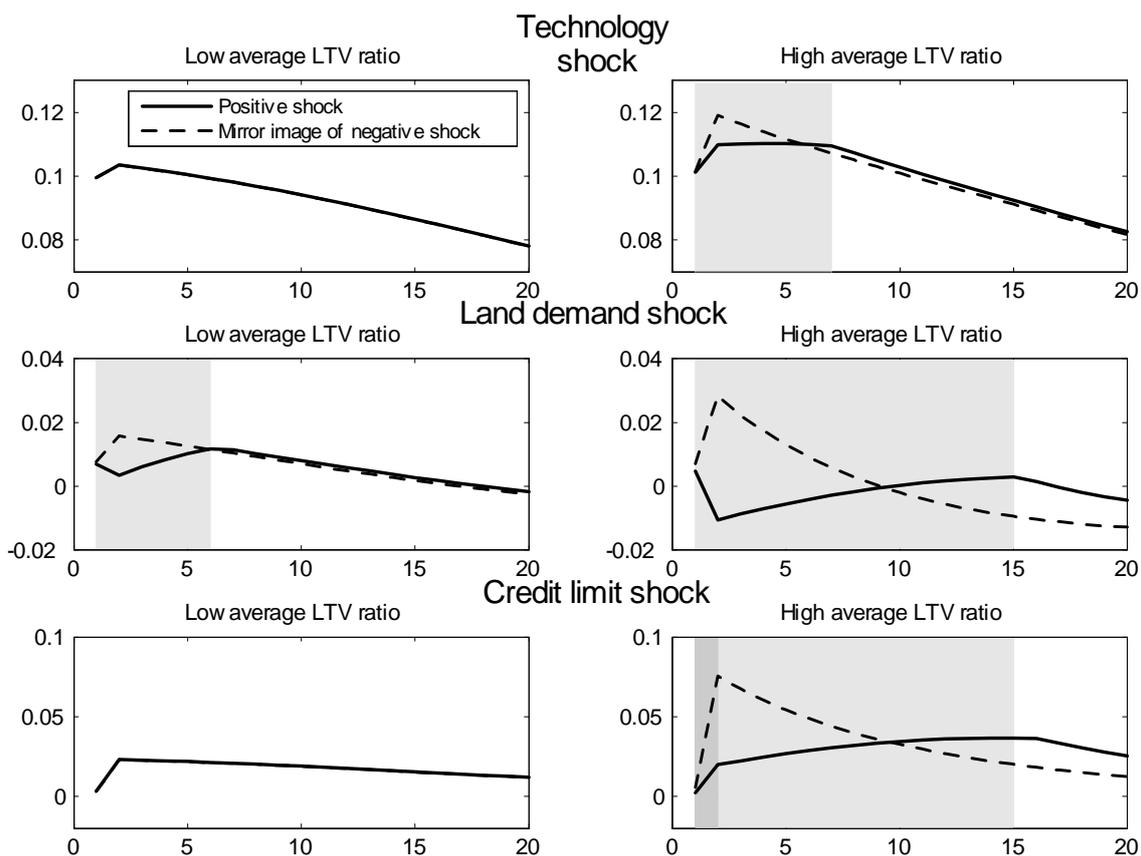
Figure 6: *Impulse responses of output to large (20 standard deviations) shocks to technology (row 1), land demand (row 2), and credit limits (row 3) for two different LTV ratios; s = 0.35 (left column) and s = 0.70 (right column).*

Notes: Light-grey periods are ones where the entrepreneurs are unconstrained; solid-grey periods are ones where all agents are unconstrained.

The second row of Figure 6 reports the response to large land demand shocks. In this case, the shock makes the entrepreneurs' collateral constraint non-binding during the first 14 quarters after the shock in the high LTV regime, while impatient households remain constrained throughout. As a result, entrepreneurs have no incentive to expand their borrowing capacity by increasing their stock of land. In fact, entrepreneurs lower their land holdings on impact, allowing patient and impatient households to increase their stock of land at the expense of non-durable consumption. In turn, the drop in land available for production leads output to contract. On the other hand, there is no attenuation of large negative shocks to the economy. In that case, both collateral constraints remain binding, giving rise to a large drop in output. The skewness emerging from large demand shocks is much weaker when the LTV ratio is low. In this case, the collateral constraint of the entrepreneur becomes non-binding for only five quarters, while the impatient household again remains constrained.

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The bottom row of Figure 6 shows the effects of large credit limit shocks. Under a high average LTV ratio, the entrepreneurs are unconstrained during the first 14 periods after a positive shock, while impatient households become unconstrained for one period. For the reasons discussed above, this leads to a muted response of output. In contrast, a large negative shock forces entrepreneurs into a sizeable deleveraging, reducing the stock of land available for production. Simultaneously, impatient households are also forced to deleverage and bring down their stock of land, which further depresses the land price, and thus the borrowing capacity of both constrained agents. The result is a large drop in output. For low LTV ratios, credit constraints remain binding throughout.

4.2   Deepening contractions

The impulse responses in the previous subsection offer a clear message: For high average LTV ratios, episodes of non-binding credit constraints are more frequent. Hence, economic contractions tend to become larger than expansions as the average LTV ratio increases, paving the way for a negatively skewed business cycle. Moreover, all three types of shock contribute to generating a more negatively skewed business cycle as the LTV ratio increases, so that their relative contribution is not crucial to our qualitative findings.
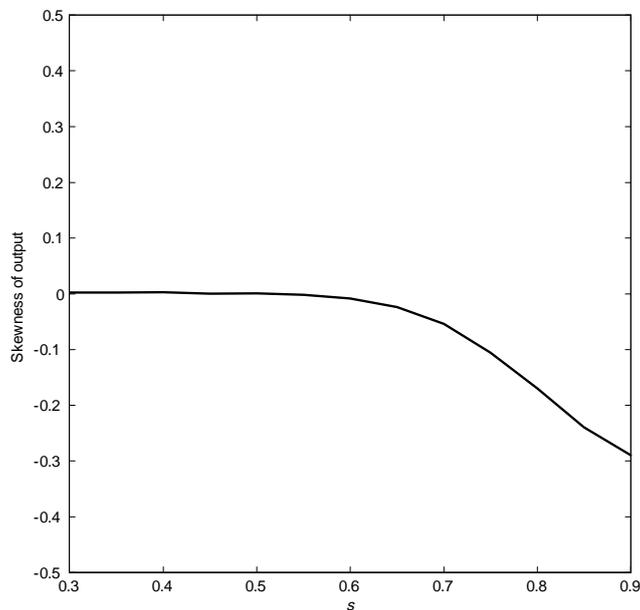

Figure 7: *Skewness of output for different LTV ratios.*

Notes: Numbers are median values from 501 stochastic model simulations of 2000 periods. All time series used to produce business cycle statistics have been preliminarily filtered.

To deepen our understanding of the properties of the model over the entire range of feasible

LTV ratios, we conduct a large set of stochastic simulations, retrieving statistics from 501 runs of 2000 periods each. Figure 7 displays the skewness of filtered output: This is practically zero going from low to intermediate values of the average LTV ratio, but becomes increasingly negative as credit limits increase further and collateral constraints become non binding more often. As illustrated in the left panel of Figure 8, the entrepreneur finds himself unconstrained as much as 50% of the time at very high LTV ratios, while impatient households only experience this instance rarely. Finally, note that the skewness of investment and aggregate consumption display a pattern similar to that of output, as it can be appreciated in the last two panels of Figure 8.[20]
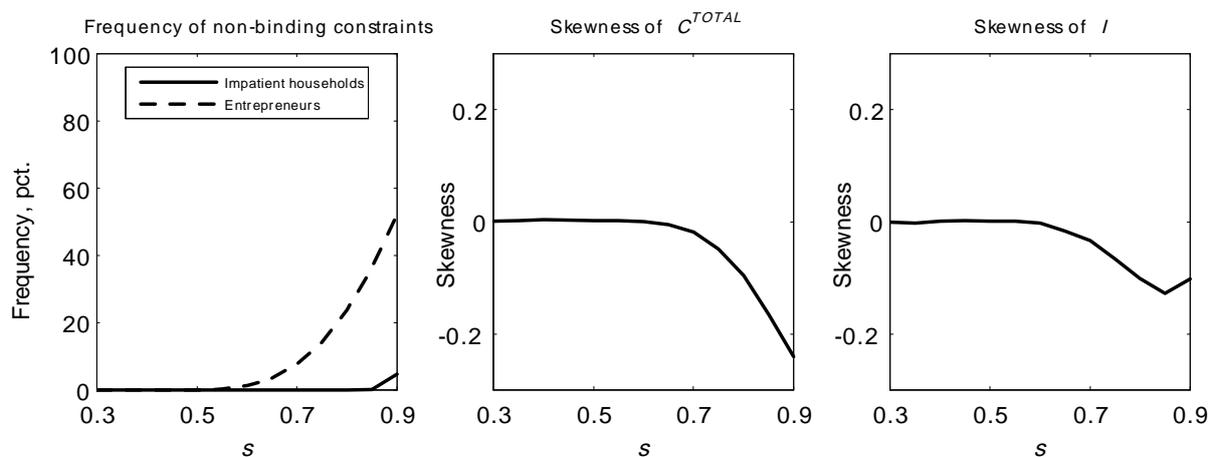

Figure 8: *Left panel: frequency of episodes of non-binding constraints for each agent. Middle and right panel: Skewness of aggregate consumption and investment for different LTV ratios.*

Notes: See the notes to Figure 7.

4.3   Skewness and volatility

The Great Moderation is widely regarded as the main development in the statistical properties of the US business cycle since the 1980s. While many have argued that the severity of the Great Recession might have marked the end of this period of relatively tranquil times, there is evidence that the US economy has not reverted back to the levels of volatility observed in the 1970s (see, e.g., Coibion and Gorodnichenko, 2011; Stock and Watson, 2012). Even more important relative to our results, recent statistical evidence has demonstrated that the Great Moderation was never associated with smaller or less frequent downturns, but has been driven exclusively by the characteristics of the expansions, whose magnitude has declined over time

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[20]In our dynamic simulations, impatient households and entrepreneurs may sometimes find themselves unconstrained even during economic downturns as a result of, e.g., a positive credit limit shock and a negative non-financial shock. In such situations—which are most likely to occur at high LTV ratios—even recessions may be dampened, thereby mitigating business cycle skewness. This explains the small reversal of investment skewness at $s = 0.90$. We return to this issue in the next subsection.

(Gadea-Rivas *et al.*, 2014, 2015). Our scope here is not to contribute to the literature on the roots of the Great Moderation, but rather to examine this major statistical development in conjunction with the change in the skewness of the business cycle, which has largely occurred over the same time span.
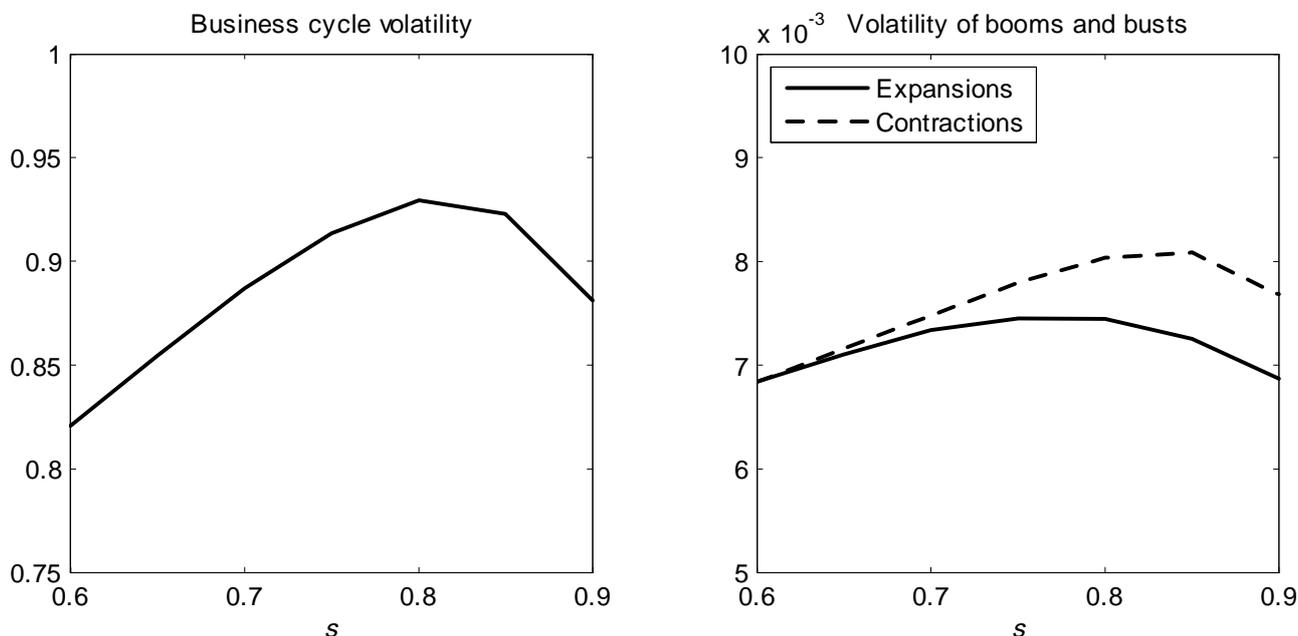

Figure 9: *Left panel: standard deviation of output. Right panel: Interquantile range for contractions (dashed line) and expansions (solid line).*

Notes: See the notes to Figure 7. In the right panel we split our simulated samples into expansions and contractions based on whether filtered output is positive or negative, after which we compute the interquartile range for each of the two subsamples.

To assess our model's ability to account for these empirical facts, the left panel of Figure 9 reports the standard deviation of output as a function of the average LTV ratio.[21] As discussed in Jensen *et al.* (2016), macroeconomic volatility displays a hump-shaped pattern in response to changes in the LTV ratio: Starting from low credit limits, higher availability of credit allows financially constrained agents to engage in debt-financed consumption and investment, as dictated by their relative impatience, thus reinforcing the macroeconomic repercussions of shocks that affect their borrowing capacity. This pattern eventually reverses, as higher LTV ratios increase the likelihood that credit constraints become non-binding. In such cases, the consumption and investment decisions of households and entrepreneurs tend to delink from changes in the value of their collateral assets, dampening the volatility of aggregate economic activity.

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[21]Having established in the previous subsection that the model displays very little skewness at average LTV ratios below $s \approx 0.6$, we focus on LTV ratios at or above this level in the remainder.

However, the volatility reversal is much stronger for positive than for negative shocks, in the face of which financial constraints tend to remain binding. This inherent property of our framework indicates that the drop in output volatility observed beyond $s \approx 0.8$ is mostly connected with expansionary periods, as in the evidence reported by Gadea-Rivas *et al.* (2014, 2015). The right panel of Figure 9 confirms this intuition: Here we compare the dispersion of expansionary and contractionary episodes, respectively, as a function of the average LTV ratio. The volatility of expansions is always lower than that of contractions, and declines over a wider range of average credit limits. The volatility of contractions, on the other hand, is declining only at the very end of the range of average LTV ratios we consider. This decline is due to the fact that at very high LTV ratios, financial constraints may sometimes be non-binding even during economic contractions in our simulations. Such situations may arise if, e.g., a negative technology shock coincides with a positive credit limit shock. This notwithstanding, the results in Figure 9 show that the empirically observed changes in the volatility of the business cycle and its skewness may be reconciled within our framework: The decline in the overall volatility of the system primarily rests on the attenuation of expansionary movements in real activity, while skewness increases as a result of the widening gap between the magnitude of expansionary and contractionary phases of the cycle, as the average LTV ratio increases. While the left panel of Figure 9 points to a hump-shaped relationship between credit limits and macroeconomic volatility, the key driver of business cycle asymmetry in our framework—occasionally non-binding credit constraints—in itself works as an impetus of lower macroeconomic volatility, *ceteris paribus*. Thus, while our analysis does not warrant the claim that the empirical developments in the volatility and skewness of the business cycle necessarily have the same origin, higher credit limits do eventually lead to a drop in the overall volatility of our model economy by making financial constraints increasingly slack.[22]

Notably, the increasing prevalence of non-binding credit constraints allows the model to account for different correlations between the volatility and the skewness of the output gap, conditional on different credit limits. Based on the comparison between Figure 7 and the left panel of Figure 9, this correlation is essentially zero up to $s \approx 0.6$, thus turning more and more negative until $s \approx 0.8$ is approached, before finally becoming positive as financial development reaches very advanced stages. These results are reminiscent of the evidence reported by Bekaert and Popov (2015), who document a positive long-run correlation between the second and third moment of output growth in a large cross-section of countries, but also a negative short-run

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[22]A large literature suggests that innovation in the credit market—especially in consumer credit and home mortgages—have played a role in the Great Moderation; see den Haan and Sterk (2010) for a review.

relationship in financially developed economies.[23]

5 Debt overhang and business cycle asymmetries

Several authors have recently pointed to the nature of the boom phase of the business cycle as a key determinant of the subsequent recession. Using data for 14 advanced economies for the period 1870–2008, Jordà *et al.* (2013) find that more credit-intensive expansions tend to be followed by deeper recessions, whether or not the recession is accompanied by a financial crisis. This evidence is consistent with the results of Mian and Sufi (2010), who identify a close connection at the county level in the US between pre-crisis household leverage and the severity of the Great Recession.

In this section we demonstrate that our model is capable of reproducing these empirical facts. To this end, Figure 10 reports the results of the following experiment: Starting in the economy's steady state, we generate a boom-bust cycle for a range of different steady-state debt levels, as reflected by different LTV ratios. In the first 5 periods, we calibrate the size of the expansionary shocks hitting the economy so as to make sure that the boom in output is identical for all the LTV ratios. In periods 6 to 10, we then feed an identical set of contractionary shock realizations into the economy. This ensures that the severity of the recession is determined by the endogenous response of the model at each different LTV ratio.[24] As the figure illustrates, the deepness of the contraction increases with the LTV ratio. A boom of a given size is followed by a more severe recession when debt levels are high, as compared with the case of scarcer credit availability. When LTV ratios are high, households and entrepreneurs are more leveraged during the boom, and they therefore need to face a more severe process of deleveraging when the recession hits. By contrast, when credit levels are low, financially constrained agents are precluded from using the credit market to shift consumption and investment forward in time during booms, and are therefore less vulnerable to contractionary shocks.

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[23] Clearly, our model cannot account for the link between the skewness and volatility of output growth in economies at early stages of their financial development. As pointed out by Bekaert and Popov (2015), while occasionally hit by crises and sudden stops, these countries experience periods of rapid economic growth that tend to generate high volatility along with positive skewness.

[24] During both the boom and the bust, the economy is hit by all three types of shocks in each period, keeping their relative sizes fixed in accordance with their standard deviations as calibrated in Subsection 3.7, but setting their persistence parameters to zero, so as to avoid that the shape of the recession may be affected by lagged values of the shocks during the boom. We make sure that impatient households and entrepreneurs remain constrained in all periods of each of the experiments reported here, so as to enhance comparability.
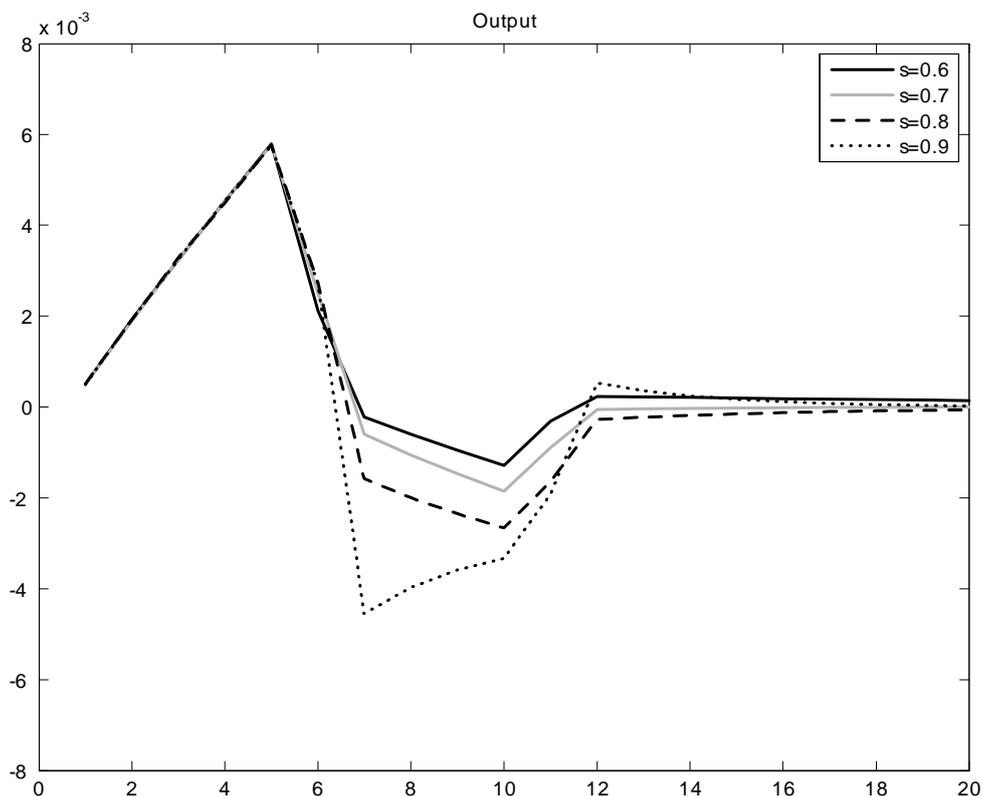
Figure 10: *Boom-bust cycles of output for different LTV ratios.*

We next focus on the nature of the boom and how this reflects into the ensuing contraction. The top left panel of Figure 11 compares the path of output in two different boom-bust cycles, while the top right panel shows the corresponding paths for aggregate debt. The dashed line represents a non-financial boom generated by a combination of technology and land demand shocks, while the solid line denotes a financial boom generated by credit limit shocks, calibrated to deliver an identical increase in output during the boom (which again lasts for the first 5 periods). As in the previous experiment, we then subject the economy to identical sets of contractionary shocks (of all three types and during periods 6-10) in each of the two cases, so as to isolate the role played by the specific type of boom in shaping the ensuing recession.[25] While the size of the booms in output is identical, the same is not the case for total debt, which displays a larger increase during the financial boom. The consequences of this show up during the subsequent contraction, which is deepest in the aftermath of the financial boom, in line with the empirical results of Jordà *et al.* (2013) and Mian and Sufi (2010).

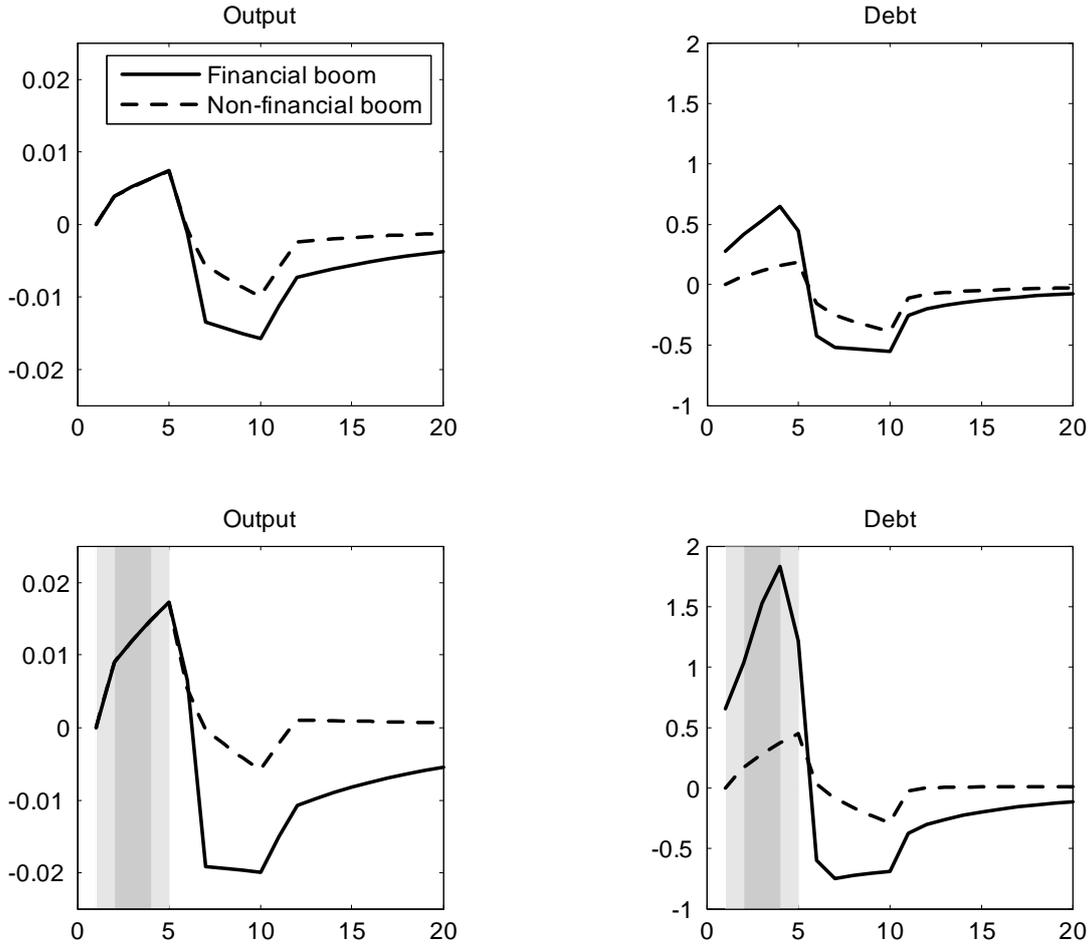[25] As in the experiment above, we set the persistence parameters of all the shock processes to zero.

Figure 11: *Boom-bust cycles of output and aggregate debt with normal shocks (top row) and large shocks (bottom row), for s = 0.70.*

Notes: Solid lines represent a financial boom, while dashed lines represent a non-financial boom. Light-grey areas denote periods where the entrepreneurs are unconstrained during financial booms; solid-grey periods are ones where all agents are unconstrained during financial booms. Both impatient households and entrepreneurs remain constrained at all times during non-financial booms.

The previous exercise confirms that the impact of constrained agents' deleveraging increases in the size of their debt. Our focus on occasionally non-binding constraints allows us to dig deeper into this point: While impatient households and entrepreneurs remain constrained at all times in the experiments reported in the top row of Figure 11, we examine the role of non-binding credit constraints in the bottom row of the figure. We do so by repeating the previous exercise for a set of larger, expansionary shocks, so that impatient households and entrepreneurs find themselves unconstrained in response to the positive financial shocks, while keeping the size and composition of the contractionary shocks during the downturn identical to those in the top row of the figure. The debt overhang narrative has even more bite in this case, as the downturn following the financial boom is now more than three times as large as that following the non-financial one. This demonstrates the importance of allowing for occasionally

non-binding credit constraints: In the bottom row of Figure 11, impatient households and entrepreneurs are (temporarily) unconstrained during the boom, but become constrained with the onset of the contraction, giving rise to a sharp deleveraging and decline in output. These findings are in line with those of Maffezzoli and Monacelli (2015), who find that the effect of a deleverage shock on output displays an S-shaped pattern with respect to the initial debt level. At low (high) levels of initial debt, a deleverage shock has a moderate effect on output, as agents remain constrained (unconstrained) before and after the shock. The largest macroeconomic effects of such shocks are observed at intermediate debt levels, when agents switch from being unconstrained to being constrained.

6   Concluding comments

We have documented a pattern of stronger negative skewness in the US business cycle over the last decades, and pointed to the concurrent increase in the LTV ratios of households and firms as a potential explanation. To substantiate this claim, we have presented a dynamic general equilibrium model with credit-constrained households and firms, in which we have shown that increasing average LTV ratios translate into a more negatively skewed business cycle, as seen in the data. This finding relies on the occasionally-binding nature of financial constraints: As LTV ratios increase, households and firms are more likely to become temporarily unconstrained during booms, while credit constraints tend to remain binding during downturns.

Our results are of interest to macroprudential policymakers for two main reasons. First, one focus of such policies has typically been to reduce LTV ratios in order to curb macroeconomic volatility. According to our findings, a reduction of the LTV ratio may have ambiguous effects on business cycle volatility. Even a policy of state-dependent LTV ratios should be carefully designed in order to properly account for the asymmetric role played by credit constraints in booms and busts. A suitable welfare analysis needs to optimally weigh these factors. This is a topic we are investigating in ongoing work. Second, our results add to a recent literature emphasizing that the seeds of the recession are sown during the boom: The nature of the boom phase, as much as its size, is an important determinant of the ensuing downturn, and policymakers should pay close attention to the build-up of credit during expansions in macroeconomic activity. Indeed, Mian et al. (2015) find that IMF and OECD forecasts made after large increases in household debt tend to overestimate subsequent output growth, and that those forecasts could be improved by adjusting them downwards to account for past increases in household as well as firm credit.

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Appendix A: Time-varying measures of volatility and skewness

Take a generic time series, y_t, so that its variance and skewness can be respectively calculated as

$$\sigma^2 = Var(y_t) = \frac{1}{T}\sum_{t=1}^{T}(y_t - \mu)^2,$$

$$\varrho = Skew(y_t) = \left\{\frac{1}{T}\sum_{t=1}^{T}(y_t - \mu)^2\right\}^{-3/2}\left\{\frac{1}{T}\sum_{t=1}^{T}(y_t - \mu)^3\right\},$$

where T denotes the number of observations in the sample and $\mu = E(y_t) = T^{-1}\sum_{t=1}^{T}y_t$ is the sample average. Define the sample autocovariance and autocorrelation as

$$\gamma_\tau = \frac{1}{T}\sum_{t=1}^{T-|\tau|}\left(y_{t-|\tau|} - \mu\right)(y_t - \mu),$$

$$\rho_\tau = \frac{\gamma_\tau}{\sigma^2}.$$

When y_t, is a Gaussian process with absolutely summable autocovariances, it can be shown that the standard errors associated to the two measures are:[26]

$$Var(\sigma^2) = \frac{2}{T}\left(\sum_{\tau=-\infty}^{\infty}\gamma_\tau\right)^2,$$

$$Var(\varrho) = \frac{6}{T}\sum_{\tau=-\infty}^{\infty}\rho_\tau^3.$$

In practice the two summations are truncated at some appropriate (finite) lag k.

The framework we follow in order to account for time-variation in the variance and skewness has a long pedigree in statistics, starting with the work of Priestley (1965), who introduced the concept of slowly varying process. This work suggests that time series may have time-varying spectral densities which change slowly over time, and proposed to describe those changes as the result of a non-parametric process. This work has more recently been followed up by Dahlhaus (1996), as well as Kapetanios (2007) and Giraitis *et al.* (2014) in the context of time-varying regression models and economic forecasting, respectively. Specifically, the time-varying variance and skewness are calculated as

$$\sigma_t^2 = Var_t(y_t) = \sum_{j=1}^{t}\omega_{j,t}(y_j - \mu_t)^2,$$

$$\varrho_t = Skew_t(y_t) = \left\{\sum_{j=1}^{t}\omega_{j,t}(y_j - \mu_t)^2\right\}^{-3/2}\left\{\sum_{j=1}^{t}\omega_{j,t}(y_j - \mu_t)^3\right\},$$

where $\mu_t = \sum_{j=1}^{t}\omega_{j,t}y_j$. Thus, the sample moments are discounted by the function $\omega_{t,T}$:

$$\omega_{j,t} = cK\left(\frac{t-j}{H}\right),$$

---

[26]The first expression computes the variance as the Newey-West variance of the squared residuals, in order to account for the autocorrelation of the errors. The second equality follows from Gasser (1975) and Psaradakis and Sola (2003).

where c is an integration constant and $K\left(\frac{T-t}{H}\right)$ is the kernel function determining the weight of each observation j in the estimation at time t. This weight depends on the distance to t normalized by the bandwidth H. Giraitis et al. (2014) show that the estimator has desirable frequentist properties. They suggest using Gaussian kernels with the optimal bandwidth value $H = T^{1/2}$.

Similarly, we can compute the time-varying standard deviation of variance and skewness estimates using a time varying estimate of the sample autocovariance and autocorrelations:

$$
\begin{aligned}
\gamma_{\tau,t} &= \sum_{j=1}^{t-|\tau|} \omega_{j,t}\left(y_{j-|\tau|}-\mu_t\right)\left(y_j-\mu_t\right), \\
\rho_{\tau,t} &= \frac{\gamma_{\tau,t}}{\sigma_t^2}.
\end{aligned}
$$

Appendix B: Assets and liabilities in the US

Figure 3 shows the ratio of liabilities to assets for households and firms in the United States, respectively. All data are taken from FRED (Federal Reserve Economic Data), Federal Reserve Bank of St. Louis. The primary source is Flow of Funds data from the Board of Governors of the Federal Reserve System. For business liabilities we use the sum of debt securities and loans of nonfinancial corporate and noncorporate businesses. As assets we follow Liu et al. (2013) and use data on both sectors' equipment and software as well as real estate at market value. For households and nonprofit organizations, we again use the sum of debt securities and loans as data for liabilities and use as assets both groups' real estate at market value and equipment and software of nonprofit organizations.[27] For the years 1945–1951, data is only available on an annual basis. For these years, we use linear interpolation to compute quarterly observations.

The ratios reported in Figure 3 are aggregate measures, and may therefore not reflect actual loan-to-value (LTV) requirements for the marginal borrower. Nonetheless, we report these figures since the flow of funds data delivers a continuous measure of LTV ratios covering the entire period 1945–2016. For households, the aggregate ratio of credit to assets in the economy is likely to understate the actual down-payment requirements faced by households applying for a mortgage loan, since loans and assets are not uniformly distributed across households. In our model, we distinguish between patient and impatient households, and we assume that only the latter group is faced with a collateral constraint. In the data, we have not made this distinction, so that the LTV ratio for households reported in Figure 3 represents an average of the LTV of patient households (savers), who are likely to have many assets and small loans, and that of impatient households (borrowers), who on average have larger loans and fewer assets. Justiniano et al. (2014) use the Survey of Consumer Finances to make this distinction, and identify borrowers as households with liquid assets of a value less than two months of their income. Based on the surveys from 1992, 1995, and 1998, they arrive at an average LTV ratio for this group of around 0.8, while our measure fluctuates around 0.5 during the 1990s. Another approach, following Duca et al. (2011), is to focus on first-time home-buyers, who are likely to fully exploit their borrowing capacity. Using data from the American Housing Survey, these authors report LTV ratios approaching 0.9 towards the end of the 1990s; reaching a peak of almost 0.95 before the onset of the recent crisis. While these alternative approaches are thus likely to result in higher *levels* of LTV ratios, we are interested in the *development over time* of these ratios. While we believe the Flow of Funds data provide the most comprehensive and consistent time series evidence in this respect, substantial increases over time in LTV ratios faced by households have been extensively documented; see, e.g., Campbell and Hercowitz (2009), Duca et al. (2011), Favilukis et al. (2015), and Boz and Mendoza (2014). It should be noted that for households, various government-sponsored programs directed at lowering the down-payment requirements faced by low-income or first-time home buyers have been enacted by different administrations (Chambers et al., 2009). These are likely to have contributed to the increase in the ratio of loans to assets illustrated in the left panel of Figure 3.

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[27]Until 2015, debt securities and loans were aggregated under the title "Credit market instruments" for businesses as well as households in the Financial Accounts of the United States.

Likewise, the aggregate ratio of business loans to assets in the data may cover for a disparate distribution of credit and assets across firms. In general, the borrowing patterns and conditions of firms are more difficult to characterize than those of households, as their credit demand is more volatile, and their assets less uniform and often more difficult to assess. Liu *et al.* (2013) also use Flow of Funds data to calibrate the LTV ratio of entrepreneurs, and arrive at a value of 0.75. This ratio is based on an assumption that commercial real estate enters with a weight of 0.5 in the asset composition of firms. In contrast, the ratio we report in Figure 3 assigns a weight of 1 to commercial real estate. While the transformation of Liu *et al.* (2013) would result in higher LTV ratios at any point in time, it would not affect the finding of rising LTV ratios over time. The secular increase in firm leverage over the second half of the 20th century has also been documented by Graham *et al.* (2014) using data from the Compustat database.[28,29] These authors report loan-to-asset ratios that are broadly in line with those we present. More generally, an enhanced access of firms to credit markets over time has been extensively documented in the literature. This involves, for instance, the emergence of a market for high-risk, high-yield bonds (Gertler and Lown, 1999), increased flexibility in firms' financing decisions, and the resulting immoderation in financial quantities (Jermann and Quadrini, 2009).

Appendix C: First-order conditions

We report the first-order conditions for each type of agent below.

Patient households

Patient households' optimal behavior is described by the following first-order conditions:

$$\frac{1}{C_t^P - \rho^P C_{t-1}^P} - \frac{\beta \rho^P}{\mathrm{E}_t\left\{C_{t+1}^P\right\} - \rho^P C_t^P} = \lambda_t^P, \tag{17}$$

$$\nu^P \left(1 - N_t^P\right)^{-\sigma_N^P} = \lambda_t^P W_t^P, \tag{18}$$

$$\lambda_t^P = \beta^P R_t \mathrm{E}_t\left\{\lambda_{t+1}^P\right\}, \tag{19}$$

$$Q_t = \frac{\varepsilon_t}{\lambda_t^P H_t^P} + \beta^P \mathrm{E}_t\left\{\frac{\lambda_{t+1}^P}{\lambda_t^P} Q_{t+1}\right\}, \tag{20}$$

where λ_t^P is the multiplier associated with (3) for $i = P$.

Impatient households

The first-order conditions of the impatient households are given by:

$$\frac{1}{C_t^I - \rho^I C_{t-1}^I} - \frac{\beta \rho^I}{\mathrm{E}_t\left\{C_{t+1}^I\right\} - \rho^I C_t^I} = \lambda_t^I, \tag{21}$$

$$\nu^I \left(1 - N_t^I\right)^{-\sigma_N^I} = \lambda_t^I W_t^I, \tag{22}$$

$$\lambda_t^I - \mu_t^I = \beta^I R_t \mathrm{E}_t\left\{\lambda_{t+1}^I\right\}, \tag{23}$$

---

[28]It should be mentioned that they also show a Flow of Funds-based measure of debt to total assets at historical cost (or book value) for firms. The increase over time in this measure is smaller. However, we believe that the ratio of debt to *pledgeable* assets at market values (as shown in Figure 3) is the relevant measure for firms' access to collateralized loans, and hence more appropriate for our purposes.

[29]We emphasize that Figure 3 reports a *gross* measure of firm leverage. Bates *et al.* (2009) report that firm leverage *net of* cash holdings has been declining since 1980, but that this decline is entirely due to a large increase in cash holdings.

$$Q_t = \frac{\varepsilon_t}{\lambda_t^I H_t^I} + \beta^I \mathrm{E}_t \left\{ \frac{\lambda_{t+1}^I}{\lambda_t^I} Q_{t+1} \right\} + s_t \frac{\mu_t}{\lambda_t^I} \frac{\mathrm{E}_t \left\{ Q_{t+1} \right\}}{R_t}, \tag{24}$$

where λ_t^I is the multiplier associated with (5) for $i = I$, and μ_t^I is the multiplier associated with (6). Additionally, the complementary slackness condition

$$\mu_t^I \left(B_t^I - s_t \frac{\mathrm{E}_t \left\{ Q_{t+1} \right\} H_t^I}{R_t} \right) = 0, \tag{25}$$

must hold along with $\mu_t^I \geq 0$ and (6).

Entrepreneurs

The optimal behavior of the entrepreneurs is characterized by:

$$\frac{1}{C_t^E - \rho^E C_{t-1}^E} - \frac{\beta \rho^E}{\mathrm{E}_t \left\{ C_{t+1}^E \right\} - \rho^E C_t^E} = \lambda_t^E, \tag{26}$$

$$\lambda_t^E - \mu_t^E = \beta^E R_t \mathrm{E}_t \left\{ \lambda_{t+1}^E \right\}, \tag{27}$$

$$\lambda_t^E = \psi_t^E \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \frac{I_t}{I_{t-1}} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right] + \beta^E \Omega \mathrm{E}_t \left\{ \psi_{t+1}^E \left(\frac{I_{t+1}}{I_t} \right)^2 \left(\frac{I_{t+1}}{I_t} - 1 \right) \right\}, \tag{28}$$

$$\psi_t^E = \beta^E r_t^K \mathrm{E}_t \left\{ \lambda_{t+1}^E \right\} + \beta^E (1 - \delta) \mathrm{E}_t \left\{ \psi_{t+1}^E \right\} + \mu_t^E s_t \frac{\mathrm{E}_t \left\{ Q_{t+1}^K \right\}}{R_t}, \tag{29}$$

$$Q_t = \beta^E r_t^H \mathrm{E}_t \left\{ \frac{\lambda_{t+1}^E}{\lambda_t^E} \right\} + \beta^E \mathrm{E}_t \left\{ \frac{\lambda_{t+1}^E}{\lambda_t^E} Q_{t+1} \right\} + s_t \frac{\mu_t^E}{\lambda_t^E} \frac{\mathrm{E}_t \left\{ Q_{t+1} \right\}}{R_t}, \tag{30}$$

where λ_t^E, μ_t^E and ψ_t^E are the multipliers associated with (9), (10), and (11), respectively. Moreover,

$$\mu_t^E \left(B_t^E - s_t \mathrm{E}_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\} \right) = 0, \tag{31}$$

holds along with $\mu_t^E \geq 0$ and (11). Finally, the definition of Q_t^K implies that

$$Q_t^K = \psi_t^E / \lambda_t^E. \tag{32}$$

Firms

The first-order conditions for the firms determine the optimal demand for the input factors:

$$\alpha \gamma Y_t / N_t^P = W_t^P, \tag{33}$$

$$(1 - \alpha) \gamma Y_t / N_t^I = W_t^I, \tag{34}$$

$$(1 - \gamma)(1 - \phi) \mathrm{E}_t \left\{ Y_{t+1} \right\} / K_t = r_t^K, \tag{35}$$

$$(1 - \gamma) \phi \mathrm{E}_t \left\{ Y_{t+1} \right\} / H_t^E = r_t^H. \tag{36}$$

Appendix D: The solution method

As discussed in the main text, we treat the collateral constraints as inequalities when we solve the model, and add two complementary slackness conditions:

$$\mu_t^I \left(B_t^I - s_t \frac{\mathrm{E}_t \left\{ Q_{t+1} \right\} H_t^I}{R_t} \right) = 0,$$

$$\mu_t^E \left(B_t^E - s_t \mathrm{E}_t \left\{ \frac{Q_{t+1}^K K_t + Q_{t+1} H_t^E}{R_t} \right\} \right) = 0.$$

We then adopt the solution method of Holden and Paetz (2012), on which this appendix builds. In turn, Holden and Paetz (2012) expand on previous work by Laséen and Svensson (2011). With first-order perturbations, this solution method is equivalent to the piecewise linear approach used in Guerrieri and Iacoviello (2014), as discussed by Guerrieri and Iacoviello (2015) and the references therein. We have verified that the solution method described in Guerrieri and Iacoviello (2015) does indeed produce identical results. Furthermore, Holden and Paetz (2012) and Guerrieri and Iacoviello (2015) evaluate the accuracy of their respective methods against a global solution based on projection methods. This is done for a very simple model with a borrowing constraint, for which a highly accurate global solution can be obtained and used as a benchmark. They find that the non-linear local approximations are very accurate. For the model used in this paper, the large number of state variables (9 endogenous state variables and 3 shocks) renders the use of global solution methods impractical due to the curse of dimensionality typically associated with such methods.

The collateral constraints put an upper bound on the borrowing of each of the two constrained agents. While the constraints are binding in the steady state, this may not be the case outside the steady state, where the constraints may be occasionally non-binding. Observe that we can reformulate the collateral constraints in terms of restrictions on each agent's shadow value of borrowing; μ_t^j, $j = \{I, E\}$: We know that $\mu_t^j \geq 0$ if and only if the optimal debt level of agent j is exactly at or above the collateral value. In other words, we need to ensure that $\mu_t^j \geq 0$. If this restriction is satisfied with inequality, the constraint is binding, so the slackness condition is satisfied. If it holds with equality, the collateral constraint becomes non-binding, but the slackness condition is still satisfied. If instead $\mu_t^j < 0$, agent j's optimal level of debt is lower than the credit limit, so that treating his collateral constraint as an equality implies that we are forcing him to borrow 'too much'. In this case, the slackness condition is violated. We then need to add shadow price shocks so as to 'push' μ_t^j back up until it exactly equals its lower limit of zero and the slackness condition is satisfied. To ensure compatibility with rational expectations, these shocks are added to the model as "news shocks". The idea of adding such shocks to the model derives from Laséen and Svensson (2011), who use such an approach to deal with pre-announced paths for the interest rate setting of a central bank. The contribution of Holden and Paetz (2012) is to develop a numerical method to compute the size of these shocks that are required to obtain the desired level for a given variable in each period, and to make this method applicable to a general class of potentially more complicated problems than the relatively simple experiments conducted by Laséen and Svensson (2011).

We first describe how to compute impulse responses to a single generic shock, e.g., a technology shock. The first step is to add independent sets of shadow price shocks to each of the two log-linearized collateral constraints. To this end, we need to determine the number of periods T in which we conjecture that the collateral constraints may be non-binding. This number may be smaller than or equal to the number of periods for which we compute impulse responses; $T \leq T^{IRF}$. For each period $t \leq T$, we then add shadow price shocks which hit the economy in period t but become known at period 0, that is, at the same time the economy is hit by the technology shock. In other words, the log-linearized collateral constraints now become:

$$\frac{nY}{n_I B^I} \widehat{B}_t^I = \widehat{s}_t + \mathrm{E}_t \left\{ \widehat{Q}_{t+1} \right\} + \widehat{H}_t^I - \beta^P \widehat{R}_t - \sum_{s=0}^{T-1} \varepsilon_{s,t-s}^{SP,I},$$

$$\frac{nY}{n_E B^E} \widehat{B}_t^E = \widehat{s}_t - \beta^P \widehat{R}_t + \frac{K}{K + QH^E}\left(\mathrm{E}_t\left\{\widehat{Q}_{t+1}^K\right\} + \widehat{K}_t\right) + \frac{QH^E}{K + QH^E}\left(\mathrm{E}_t\left\{\widehat{Q}_{t+1}\right\} + \widehat{H}_t^E\right) - \sum_{s=0}^{T-1}\varepsilon_{s,t-s}^{SP,E},$$

where $\varepsilon_{s,t-s}^{SP,j}$ is the shadow price shock that hits agent j in period $t = s$, and is anticipated by all agents in period $t = t - s = 0$ ensuring consistency with rational expectations. We let all shadow price shocks be of unit magnitude. We then need to compute two sets of weights α_{μ_I} and α_{μ_E} to control the impact of each shock on μ_t^I and μ_t^E. The 'optimal' sets of weights ensure that μ_t^I and μ_t^E are bounded below at exactly zero. The weights are computed by solving the following quadratic programming problem:

$$
\begin{aligned}
\alpha^* &\equiv \begin{bmatrix} \alpha_{\mu_I}^{*\prime} & \alpha_{\mu_E}^{*\prime} \end{bmatrix}' \\
&= \arg\min \begin{bmatrix} \alpha_{\mu_I}' & \alpha_{\mu_E}' \end{bmatrix}\left[\begin{bmatrix} \mu^I + \widetilde{\mu}^{I,A} \\ \mu^E + \widetilde{\mu}^{E,A} \end{bmatrix} + \begin{bmatrix} \widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}} \end{bmatrix}\begin{bmatrix} \alpha_{\mu_I} \\ \alpha_{\mu_E} \end{bmatrix}\right],
\end{aligned}
$$

subject to

$$\alpha_{\mu_j}' \geq 0,$$

$$\mu^j + \widetilde{\mu}^{j,A} + \widetilde{\mu}^{j,\varepsilon^{SP,j}}\alpha_{\mu_j} + \widetilde{\mu}^{j,\varepsilon^{SP,k}}\alpha_{\mu_k} \geq 0,$$

$j = \{I, E\}$. Here, μ^j and $\widetilde{\mu}^{j,A}$ denote, respectively, the steady-state value and the unrestricted relative impulse response of μ^j to a technology shock, that is, the impulse-response of μ^j when the collateral constraints are assumed to always bind. In this respect, the vector $\begin{bmatrix} \mu^I + \widetilde{\mu}^{I,A} \\ \mu^E + \widetilde{\mu}^{E,A} \end{bmatrix}$ contains the absolute, unrestricted impulse responses of the two shadow values stacked. Further, each matrix $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$ contains the relative impulse responses of μ^j to shadow price shocks to agent k's constraint for $j, k = \{I, E\}$, in the sense that column s in $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$ represents the response of the shadow value to a shock $\varepsilon_{s,t-s}^{SP,j}$, i.e. to a shadow price shock that hits in period s but is anticipated at time 0, as described above.[30] The off-diagonal elements of the matrix $\begin{bmatrix} \widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}} \end{bmatrix}$ take into account that the impatient household may be affected if the collateral constraint of the entrepreneur becomes non-binding, and *vice versa*. Following the discussion in Holden and Paetz (2012), a sufficient condition for the existence of a unique solution to the optimization problem is that the matrix $\begin{bmatrix} \widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}} \end{bmatrix} + \begin{bmatrix} \widetilde{\mu}^{I,\varepsilon^{SP,I}} & \widetilde{\mu}^{I,\varepsilon^{SP,E}} \\ \widetilde{\mu}^{E,\varepsilon^{SP,I}} & \widetilde{\mu}^{E,\varepsilon^{SP,E}} \end{bmatrix}'$ is positive definite. We have checked and verified that this condition is in fact always satisfied.

We can explain the nature of the optimization problem as follows. First, note that $\mu^j + \widetilde{\mu}^{j,A} + \widetilde{\mu}^{j,\varepsilon^{SP,j}}\alpha_{\mu_j} + \widetilde{\mu}^{j,\varepsilon^{SP,k}}\alpha_{\mu_k}$ denotes the combined response of μ_t^j to a given shock (here, a technology shock) *and* a simultaneous announcement of a set of future shadow price shocks for a given set of weights. Given the constraints of the problem, the objective is to find a set of optimal weights so that the impact of the (non-negative) shadow-price shocks is exactly large enough to make sure that the response of μ_t^j is never negative. The minimization ensures that the impact of the shadow price shocks will never be larger than necessary to obtain this. Finally, we only allow for solutions for which the value of the objective function is zero. This ensures that at any given horizon, positive shadow price shocks occur if and only if at least one of the two constrained variables, μ_t^I and μ_t^E, are at their lower bound of zero in that period. As pointed out by Holden and Paetz (2012), this can be thought of as a complementary slackness condition on the two inequality constraints of the optimization problem. Once we have solved the minimization problem, it is straightforward to compute the bounded impulse responses of all endogenous variables by simply adding the optimally weighted shadow price shocks to the unconstrained impulse responses of the model in each period.

We rely on the same method to compute dynamic simulations. In this case, however, we need to

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[30]Each matrix $\widetilde{\mu}^{j,\varepsilon^{SP,k}}$ needs to be a square matrix, so if the number of periods in which we guess the constraints may be non-binding is smaller than the number of periods for which we compute impulse responses, $T < T^{IRF}$, we use only the first T rows of the matrix, i.e., the upper square matrix.

allow for more than one type of shock. For each period t, we first generate the shocks hitting the economy. We then compute the unrestricted path of the endogenous variables given those shocks and given the simulated values in $t-1$. The unrestricted paths of the bounded variables (μ_t^I and μ_t^E) then take the place of the impulse responses in the optimization problem. If the unrestricted paths of μ_t^I and μ_t^E never hit the bounds in future periods, our simulation for period t is fine. If the bounds are hit, we follow the method above and add anticipated shadow price shocks for a sufficient number of future periods. We then compute restricted values for all endogenous variables, and use these as our simulation for period t. Note that, unlike the case for impulse responses, in our dynamic simulations not all anticipated future shadow price shocks will eventually hit the economy, as other shocks may occur before the realization of the expected shadow price shocks and push the restricted variables away from their bounds.

# Appendix E: Baseline parameter values

We parameterize the model to match the quantitative characteristics of the U.S. business cycle. We interpret one period as a quarter. Therefore, we set $\beta^P = 0.99$, implying an annualized steady-state rate of interest of about 4%. Moreover, we have assumed that impatient households and entrepreneurs have lower discount factors than patient households. In the ballpark of available estimates, we set $\beta^I = \beta^E = 0.97$, implying a rather conservative choice about the relative impatience of borrowers and lenders. The Frisch elasticity of labor supply is given by the inverse of φ^i times the steady-state ratio of leisure to work. Calibrating the latter to around 3 for both types of households, a Frisch elasticity of labor supply of $1/3$ implies $\varphi^i = 9$, $i = \{P, I\}$. We use $\nu^i = 0.27$ for $i = \{P, I\}$, in order to ensure that patient households work about $1/4$ of their time in steady state, and impatient households slightly more. We calibrate the model so as to obtain a steady-state ratio of residential land to output around 1.45, and of commercial land to output around 0.65, both at the annual level, following values reported by Liu *et al.* (2013). This requires a value of $\varepsilon = 0.085$.

As to the production technology, we set $\gamma = 0.7$, implying a non-labor share in the production function slightly below $1/3$. We set the labor income share of patient households to $\alpha = 0.7$, in line with available estimates: Iacoviello (2005) obtains an estimate of 0.64 by matching impulse responses from his model to those from a VAR, while Iacoviello and Neri (2010) find a value of 0.79 using Bayesian estimation. The parameter ϕ, which multiplied by $(1-\gamma)$ measures land's share of inputs, is set to 0.13, somewhat higher than the estimated value from Liu *et al.* (2013). We assume a capital depreciation rate of $\delta = 0.035$. The implied annual ratio of capital to output is around 1.15, as in Liu *et al.* (2013). For the investment adjustment cost parameter, Ω, empirical estimates from estimated general equilibrium models range from nearly to zero in Liu *et al.* (2013) to above 10 in Christiano *et al.* (2014). We choose an intermediate value of $\Omega = 4$. In ongoing work, we attempt to estimate the degree of habit formation, ρ^i. So far, we simply abstract from habit formation altogether, i.e., we set $\rho^i = 0$, $i = \{P, I, E\}$.

For the technology shock, we choose values similar to those applied in most of the real business cycle literature, $\rho_A = 0.97$ and $\sigma_A = 0.005$ (see., e.g., Mandelman *et al.*, 2011). These values are largely in line with those obtained from recently estimated DSGE models; see, e.g., Jermann and Quadrini (2012) and Iacoviello (2015). For the land demand shock, we set $\rho_\varepsilon = 0.98$, reflecting the high degree of persistence of this shock found by Liu *et al.* (2013), and $\sigma_\varepsilon = 0.06$, in line with these authors as well as Iacoviello and Neri (2010) and Iacoviello (2015). For the credit limit shock, we set the persistence parameter $\rho_s = 0.98$, in line with estimated coefficients from univariate regressions of the LTV-series displayed in the Introduction. We then calibrate σ_s to obtain a standard deviation of the process for $\log s_t - \log s$ of around 0.06, as estimated by Liu *et al.* (2013). This implies $\sigma_s = 0.0119$. These values are very close to those reported by Jermann and Quadrini (2012), while Iacoviello (2015) finds a somewhat lower persistence.

Appendix F: Additional figures

Figure D.1. Cyclical components in the LTV ratios of households and firms.
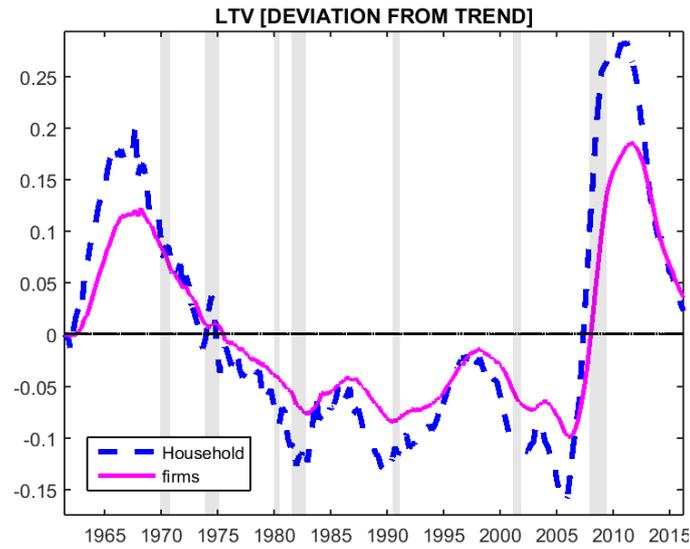

Notes. The continuous (dashed) line graphs the cyclical component of households' (firms') LTV ratio obtained through the decomposition employed in Figure 1. The vertical shadowed bands denote the NBER recession episodes.

Figure D.2. The ratio of liabilities-to-assets for households and firms in the US.
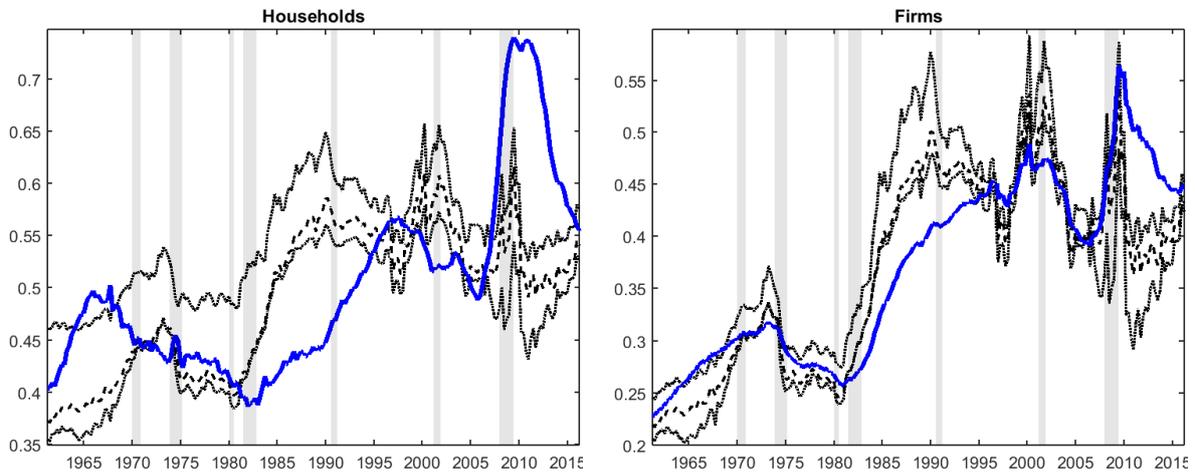
Notes. The continuous line graphs the ratio of liabilities-to-assets for households and firms, obtained from Flow of Funds data of the United States. The dashed line denotes the underlying long-run trend of the original series, obtained through a Beveridge and Nelson (1981) decomposition. The dotted lines represent the 68% confidence bands, calculated with 1000 bootstrap replications. The vertical shadowed bands denote the NBER recession episodes.

Figure D.3. US states ordered by households' average debt-to-income ratio.
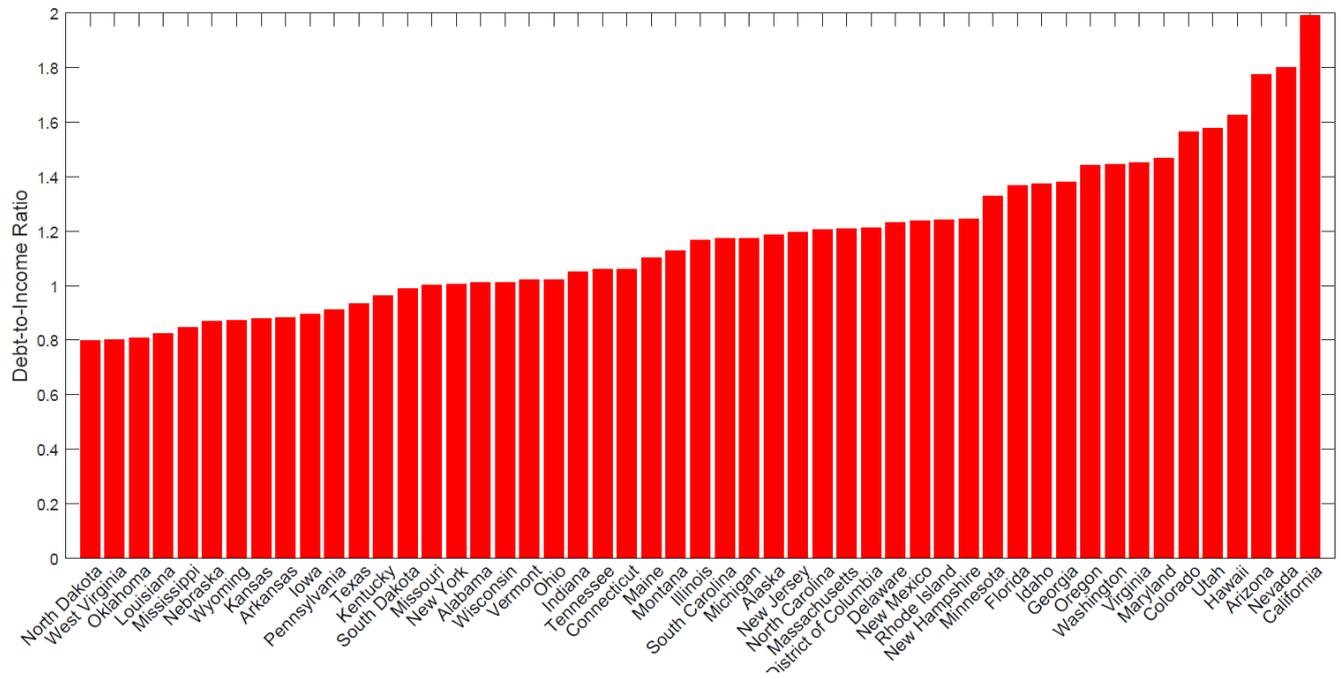
Notes. The US States are ordered by the average debt-to-income ratio in the household sector, over the period 2003-2007.